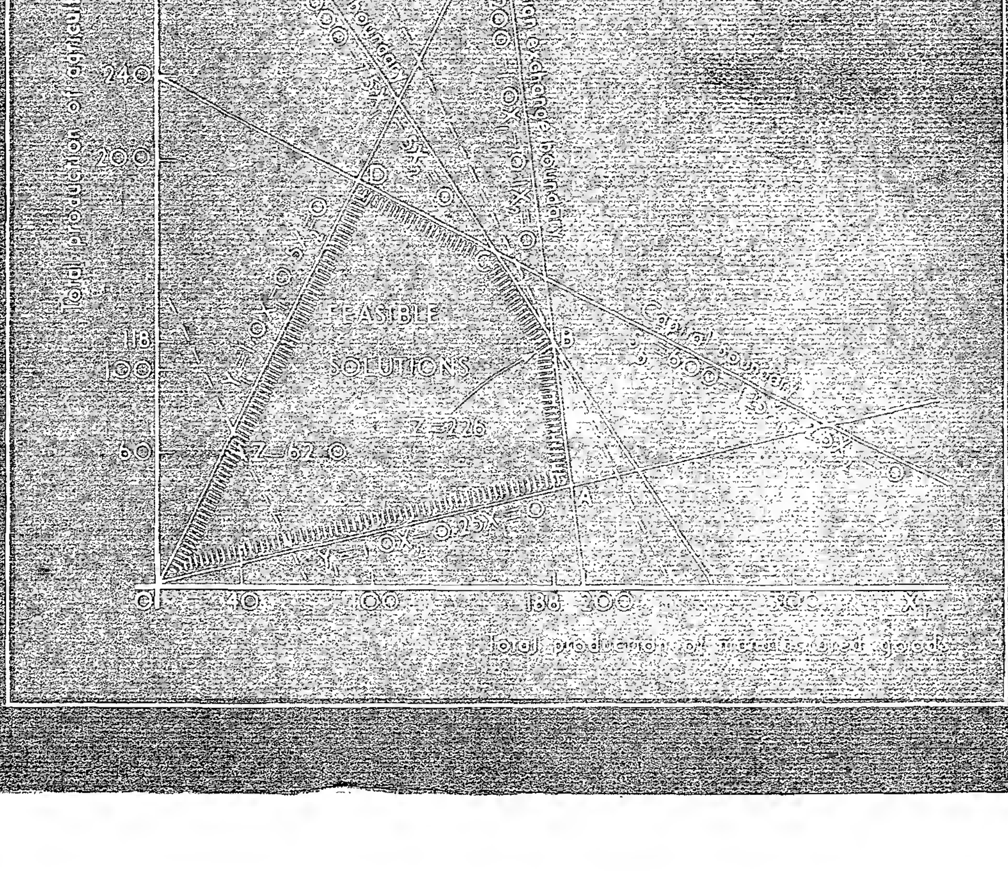
DEVELOPMENT PLANNING
MODELS AND METHODS

MICHAEL P. TODARO



DEVELOPMENT PLANNING

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Development Planning

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MICHAEL P. TODARO

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Preface

Throughout the less developed nations of the world the quest for rapid economic progress has been predicated largely upon the formulation and implementation of comprehensive development plans. Planning has become a vital instrument in the strategy of modernization. Ministries of economic planning in these nations are regularly engaged in the process of drawing up such plans in order to set forth in a logical and consistent manner the priorities, goals, and aspirations of their governments. Unfortunately, undergraduate students in these countries often find it very difficult to fully comprehend the internal mechanisms of the planning models which provide the basis for the achievement of the varied production, consumption, and resource utilization targets.

The primary purpose of this book is to introduce these undergraduate students to the most common and widely used economic models of development planning in poor nations. A secondary objective is to give students a feel for the 'thought processes' involved in the formulation of a development plan by actually working step by step through hypothetical planning problems. The emphasis is on plan formulation rather than plan implementation. Particular stress is placed on the input-output model, both static and dynamic, as a tool of comprehensive and internally consistent planning. No mathematical background beyond elementary algebra is required. In fact, the vast majority of examples utilize nothing beyond simple arithmetic techniques.

This short book will, hopefully, make a useful contribution to undergraduate courses in Economic Development and Development Planning, especially when used in conjunction with books such as W. Arthur Lewis's *Development Planning: The Essentials of Economic Policy*, and Albert Waterston's *Development Planning: Lessons of Experience,* which concentrate more exclusively on problems of plan implementation as well as the institutional basis of plan formula-

tion. Its brevity and relative simplicity can also make it a useful reference work for the interested non-academic reader.

The author would like to express his gratitude to the many staff members of the Institute for Development Studies in Nairobi who read through earlier drafts of the manuscript and offered constructive and useful suggestions. In particular, Professor John R. Harris of MIT made many valuable suggestions for improvement. The author has a special feeling of indebtedness to Professor Philip W. Bell for his earlier encouragement to include such a text on development planning as part of a larger project initiated at Makerere University to provide teaching materials that would be meaningful and relevant to the students in African universities. Finally, a sincere expression of gratitude goes to my wife, Elaine, for the many tireless hours she spent typing, proofreading, and editing the manuscript.

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March, 1971

MICHAEL P. TODARO

CHAPTER 1

Accelerating the Pace of Economic Growth: THE ROLE OF DEVELOPMENT PLANNING

THE NATURE OF ECONOMIC PLANNING

The two decades since 1950 have been marked by the emergence of less developed nations as a growing political and economic force in international affairs. The rising aspiration of 'Third World' countries to catch up economically with the advanced industrial nations as rapidly as possible has been reflected in the almost universal acceptance of development planning as the principal means towards the achievement of accelerated growth. Planning has now become an accepted fact of the economic life of most contemporary developing nations. It is the purpose of this book to explain and elucidate by means of simple hypothetical examples the 'thought processes' of a typical development plan and the economic models most commonly utilized in the formulation of these plans.

Economic planning may be described as the conscious effort of a central organization to influence, direct, and, in some cases, even control changes in the principal economic variables (e.g. GDP, consumption, investment, saving, etc.) of a certain country or region over the course of time in accordance with a predetermined set of objectives. The essence of economic planning is summed up in these notions of *influence, direction, and control*. Similarly, we can describe an *economic plan* as a specific set of quantitative economic targets to be reached in a given period of time. Economic plans may be either comprehensive or partial. A comprehensive plan sets its targets to cover all major aspects of the national economy. A partial plan covers only a part of the national economy, e.g. industry, agriculture, the public sector, the foreign sector, and so forth.

Proponents of economic planning in developing countries argue that the uncontrolled market economy can, and often does, subject these nations to

economic stagnation, fluctuating prices, and low levels of employment. In particular, they claim that the market economy is not geared to the principal operational task of poor countries, namely, how to mobilize limited resources in a way that will bring about the necessary structural change so as to stimulate smooth, progressive and balanced growth of the entire economy. Planning has come to be accepted, therefore, as an essential and pivotal means of guiding and accelerating economic growth in underdeveloped countries. However, before we embark in the following chapters upon our analysis of some of the planning techniques utilized in developed as well as less developed nations, let us first discuss further the various aspects and characteristics of economic planning as it is practised in different situations and at different stages of economic maturity.

THE DIFFERENTIAL ROLE OF PLANNING IN THE WORLD'S ECONOMIES

Unfortunately, the usual division of economic systems into market economies and planned economies can be very misleading when viewed in an overall perspective. In the real world there are no completely planned or completely unplanned economies; planning is obviously a matter of degree. For example, an analysis of the decentralized socialist economy within the context of a market system is legitimate to the extent that in this type of socialist economy prices are determined by the forces of market supply and market demand and such prices play a key role in the overall allocation of resources. Consequently, from the static viewpoint of optimal resource allocation and economic efficiency, decentralized socialism can be said to belong in the market category. However, to the extent that the level of savings is corporately determined by a central planning board and is consciously set aside to finance investment in future periods, the dynamic aspects of the decentralized socialist system indicate a close affinity with the planned economy.[1] Therefore, in order to avoid any further confusion, let us distinguish among three fundamental but distinct degrees of economic planning.

PLANNING IN CAPITALIST ECONOMIES

First, we must recognize that even in predominantly private enterprise economies like those of the United States, the United Kingdom, and Japan, planning plays a vital although relatively indirect role in the economic process. In the context of these economies, planning usually consists in the conscious effort by the Government to attain rapid economic growth with high employment and

[1] See P.W. Bell and M.P. Todaro, *Economic Theory: An Integrated Text with Special Reference to Tropical Africa and Other Developing Areas* (Nairobi, Oxford University Press, 1969), chapter 13, for an analysis of the decentralized socialist economy.

stable prices through its various fiscal and monetary policies. Recognizing that the completely unfettered play of the market mechanism can lead to highly unstable situations reflected in severe fluctuations of income and employment over the course of business cycles, these governments actively attempt to create conditions that will prevent economic instability while still stimulating economic growth. The principal policy instruments used are primarily those in the fields of monetary, fiscal, and foreign trade relations. Greater employment and higher incomes for a growing population are induced by expansionary monetary policy, increased government spending and tax rate adjustments. Inflation and deflation are held in check by counter-cyclical fiscal policies, interest rate adjustments, and wage-price guidelines. Balance of payments fluctuations are counteracted by tariff adjustments, exchange controls, import quotas and tax incentives. In all of the above cases, however, the instruments of policy are *active* but *indirect*. They are active to the extent that they push the economy in a desired direction. They are indirect in the sense that they are intended merely to create favourable conditions in which private decision makers will be influenced to behave in a manner conducive to the continuous realization of stable economic growth. Although no detailed economic plan in the sense of a set of specific targets is drawn up in most capitalist economies, limited government planning is nevertheless carried out on the basis of analysis of past trends and projections of future economic conditions.

PLANNING IN COLLECTIVIST ECONOMIES

The second category of economic planning is associated mainly with the Soviet Union and those Soviet-type economies of Eastern Europe and Asia where the government *actively* and *directly controls* the movements of the economy through a centralized decision-making process. A specific set of targets predetermined by central planners forms the basis upon which a complete and comprehensive national economic plan is drawn up. Resources, both material and financial, are allocated not on the basis of market prices and conditions of supply and demand as in market economies but rather in accordance with the material, labour and capital requirements of the overall plan. Thus, the essential difference between planning in capitalist and collectivist economies is one of *inducement* versus *control*. While the former merely attempts to prevent the economy from straying off a desired path of stable growth by active but indirect instruments of policy, the latter not only draws up a specific set of targets representing a desired course of economic progress but it also attempts to *implement* its plan by directly controlling the activities of practically all productive units in the entire national economy. In short, the collectivist economic plan dictates the future position of all economic variables. However, not even the economies of communist countries are one hundred per cent centrally planned. For example, in the Soviet Union, aspects of the market economy are

becoming increasingly evident in the production, distribution and pricing of a wide variety of consumer goods. But we shall reserve detailed discussion of the Soviet economy for the latter half of chapter 3.

PLANNING IN 'MIXED' ECONOMIES

Finally, the third, and for our purposes, the most important example of economic planning lies in the realm of development planning within the framework of the 'mixed' economies of the African variety. These economies are characterized by the existence of an institutional setting in which part of the productive resources are privately owned and operated while the other part belongs to the public sector. The actual proportionate division of public and private ownership varies from country to country. However, unlike capitalist economies where there usually exists only a small degree of public ownership, African economies are distinguished by a substantial amount of government influence. The private sector of the economy commonly consists of three distinct forms of individual ownership: (1) the traditional subsistence sector consisting of small scale private farms and handicraft shops selling a part of their produce to local markets, (2) medium sized capitalist enterprises in agriculture, industry, trade, and transport owned and operated by African, and in some countries, Asian entrepreneurs (*dukas*, small plantations and factories, cinema theatres, restaurants, furniture shops and individually owned lorries and taxis provide the usual example of this second type of private enterprise), and (3) expatriate enterprises and large scale plantations primarily catering to foreign markets. The capital for such enterprises usually comes from abroad while a good proportion of profits tend to be transferred overseas.

In the context of such an institutional setting we can recognize two principal aspects of development planning in 'mixed' economies:

1. the government's deliberate utilization of domestic saving and foreign finance to carry out public investment projects and to mobilize and channel scarce resources into areas that can be expected to make the greatest contribution towards the realization of long term economic progress (e.g. the construction of railroads, schools, hydroelectric projects, and other components of 'economic infrastructure', as well as the creation of import-substituting industries), and
2. governmental policy to facilitate, stimulate, direct, and in some cases, even control private economic activity in order to ensure a harmonious relationship between the desires of private businessmen and the economic plans of the central government.

The compromise nature of this situation between capitalist inducement and collectivist control is readily evident from the above characteristics of planning in mixed economies. Since most of the newly independent African countries fall into this third category, let us briefly look now at some of the economic

and institutional conditions in tropical Africa, as indeed in most less developed countries, that have led many of the world's economists to conclude that some degree of planning is necessary if these countries are to overcome the debilitating forces of 'poverty, ignorance, and disease'.

THE RATIONALE OF PLANNING IN AFRICAN ECONOMIES

The adoption of the techniques of economic planning in Africa as elsewhere is necessitated by a number of similar institutional and socio-economic conditions of which the following are most readily discernible:

1. *Absence of well organized markets.* Markets in Africa are permeated by imperfections both of structure and operation. Commodity and factor markets are poorly organized and fail to provide the necessary information to permit consumers and producers to act in a way that is conducive to efficient production and distribution. Secondly, well organized capital markets based on the existence of specialized financial institutions performing a great variety of monetary functions (such as the channelling of savings into loan markets to provide necessary capital funds to finance investment projects) are either non-existent or poorly developed in African countries. In short, the inefficiency or absence of well organized commodity, factor, and capital markets reduces considerably the ability of the economic system to function effectively without some form of external interference.
2. *Need for rapid institutional transformation.* Economic development is a necessary prerequisite for self-sustaining material growth. In tropical Africa economic development is associated with the notion of structural and institutional change, i.e. with the total transformation of an entire country from a traditional, subsistence, hoe-agricultural society to a modern, monetary, industrially self-sufficient economy. If such a transformation is not to lead to chaos and ultimate failure, a step by step detailed programme of action which takes account of all non-economic as well as economic implications of each and every individual project must be drawn up. The economic plan serves as a blueprint for action in the pursuit of economic growth, institutional reconstruction, and the attainment of the ideals of African Socialism.
3. *Necessity of allocating scarce resources into the most productive channels.* African economies cannot afford to waste their limited financial and skilled manpower resources on unproductive ventures. Investment projects must be chosen not only on the basis of a partial productivity analysis dictated by individual capital/output ratios but rather in the context of an overall development programme which takes account of external economies, indirect repercussions, and long term objectives. Skilled manpower must also be utilized where its contribution will be most widely felt. Economic

planning helps to modify the restraining influence of limited resources by recognizing the existence of particular constraints and by choosing and co-ordinating investment projects so as to channel these scarce factors into their most productive outlets.

4. *Psychological impact of a programme of national objectives.* Finally, we should not fail to recognize that a detailed statement of national economic goals and objectives in the form of a specific development plan can have a tremendous psychological impact. It can often succeed in rallying the people behind the government in a national campaign to eliminate poverty. By mobilizing popular sentiment and cutting across tribal factions with the plea to all citizens to 'work together', an enlightened central government, through its economic plan, can provide the needed incentive to overcome the inhibiting forces of traditionalism in the quest for widespread material progress.

THE USE OF MATHEMATICAL AND STATISTICAL MODELS FOR ECONOMIC PLANS

Economic plans are commonly drawn up in accordance with some particular mathematical or statistical model thought to be most representative of the unique structural conditions existing in an economy during the proposed planning period. Consequently, let us begin by considering some important formal characteristics of all economic models.

THE NATURE OF AN ECONOMIC MODEL

An economic model may be defined as an organized set of relationships that describes the functioning of an economic entity, whether it be the individual household or firm, the national economy, or the world economy, under a set of simplifying assumptions. All economic reasoning is ultimately based on models although in many instances no quantitative expressions may actually be used. In our analysis of the behaviour of households and firms we are implicitly using economic models that employ relationships between indifference curves and budget constraints, production isoquants and cost-outlay lines, under the simplifying assumptions of utility and profit maximization. In addition to the partial and general equilibrium models of 'micro' economic analysis, we commonly utilize Keynesian and other aggregate, or 'macro' economic models to explain the determinants of income and employment in a national economy. Finally, there are a number of models of international trade used to express conditions for the stability and instability of the world economy. As we turn now to an examination of planned economies we shall discover that economic models are probably even more helpful in this particular field not only because

of their analytic and explicative value but primarily because of their practical usefulness.

In the context of planning, economic models provide a logically systematic and internally consistent operational framework based on an important set of structural interrelationships in the economy. These interrelationships can be expressed in meaningful quantitative terms and then formulated as a guiding programme for future development. The choice of a particular model depends upon a number of criteria. The most important of these include the existing stage of realized development, the availability of adequate statistics and the role of the government in the economic process. However, the formal, structural characteristics of all economic models from the simplest 'micro' model to the most complex comprehensive planning model are identical with respect to three essential integral features of any mathematical or statistical model.

THREE BASIC COMPONENTS OF A MODEL

All economic models consist of the following three basic structural elements; (1) a set of variables, (2) a list of fundamental relationships, and (3) a number of strategic coefficients.

The choice of *variables* to be included in any model naturally depends upon the use to which the model is being put. Variables may be either independent (exogenous) or dependent (endogenous). For example, when considering the simple Keynesian consumption function, income is the independent (exogenous) variable while the level of consumption is the dependent (endogenous) variable. In consumer demand models, the quantity demanded of a particular good (dependent variable) is usually said to depend on the price of that product, the prices of related commodities, the consumer's disposable income, and where quantitatively expressable, the consumer's preference patterns—all the preceding, of course, constituting the independent variables of this particular model. If we wished to construct a model to analyse the determinants of income and employment for a national economy we would have to include such crucial variables as consumption, investment, government expenditure, the money supply, the interest rate, the money wage rate, an index of prices, and other relevant magnitudes. Similarly, when formulating development plans with the aid of an economic model, we shall have to isolate those strategic variables such as investment, domestic saving, government finance, foreign exchange, and the supply of skilled manpower which together constitute the major determinants and constraints on the growth and stability of less developed nations.

The functional relationships of a model link the independent and dependent variables together in a set of specific structural equations. Each equation represents a unique causal relationship showing the particular manner in which

the movement of one or more independent variables affects the quantitative value of the dependent variable.

The precise causal link between variables is expressed by the *coefficients* of the model. Coefficients describe the intensity with which one variable affects, by means of its particular causal relationship, the value of another variable. If, for example, changes in imports, ΔM, are expressed as a simple linear function of changes in the level of national income, ΔY, in the following manner:

$$\Delta M = m\,\Delta Y$$

or, specifically,

$$\Delta M = 0.40\,\Delta Y$$

where the coefficient $m=0.40$ is the average (marginal) propensity to import, the above equation would state that 40 cents out of every additional shilling of income will be spent on imports. Had m been equal to some numerical value other than 0.40, say 0.25, then this simplified one-equation import model would yield different results.

Once the important variables are isolated, functional relationships are specified and coefficients are estimated, the complete economic model provides a systematic and internally consistent, logical, mathematical framework, which enables its user to analyse a particular problem with a considerable degree of rigour and precision.

As an illustration of a complete economic model with which the student is well familiar, let us briefly look at the simple Keynesian model of income determination. This model can be represented by the following three equations:

$$(1.1)\quad Y = C + I$$

$$(1.2)\quad C = a + b\,Y$$

$$(1.3)\quad I = \mathring{I}$$

which reduce to a single equation:

$$Y = a + \mathring{b}\,Y + I$$

or

$$(1.4)\quad Y = \frac{1}{1-b}(a + \mathring{I}).$$

Letting $k = \dfrac{1}{1-b}$ we obtain a final expression:

$$(1.5)\quad Y = k\,(a + \mathring{I}).$$

Equation (1.5) represents a simplified model of income determination which consists of the following three components:

1. an independent (known) variable $\mathring{I}$ and a dependent (unknown) variable Y,
2. a specific functional relationship expressing the nature of the causal link between $\mathring{I}$ and Y, and

3. a coefficient k, the investment multiplier, which describes the intensity of the functional relationship.[2]

THE MODEL AS A FRAMEWORK FOR THE FORMULATION OF CONSISTENT ECONOMIC PLANS

The construction of a mathematical model geared to the particular institutional characteristics of a developing economy can be a great aid to the formulation of a consistent and manageable development plan. The technical relationship between the plan and the model might be expressed as follows: the figures which will ultimately form the detailed components of the plan are the values of the dependent or endogenous variables of the model.[3] These variables will constitute the outcome, or, mathematically speaking, the solutions of the system of equations, given the exogenous or independent variables. The use of mathematical models, therefore, has the advantage of providing a systematic framework within which one can attack particular planning problems on the basis of the best available evidence about the unique structural relationships that prevail in a given economy.

In addition to these desirable attributes of logic and systematism, mathematical models also possess the dual advantage of clarity and consistency. Their clarity is derived from the manner in which they are effectively able to reveal the intricate economic interdependences operative in the economy in the form of, say, a simple set of linear equations which, though formidable in appearance, are often mathematically quite unsophisticated. By isolating the crucial variables and measuring their influence on the movements of the economy over the course of the planning period, economic models can strike at the heart of the development problem and ease the analytic burden of the planning authorities. Moreover, a well constructed model can greatly help to overcome any of the inconsistencies or inner contradictions that often arise in a comprehensive development plan by seeing to it that the combined projects constituting the overall plan do not require the use of more material, capital and skilled labour resources than will be available during the planning period. In short, mathematical models can be utilized to check the *feasibility* of a particular plan and to ensure *optimality* in the use of available information and limited resources.

[2] The additive constant, a, also forms part of this model but it has no effect on the solution when changes in the magnitudes of the variables are considered.

[3] For example, the Harrod-Domar model is often used to provide a first approximation of the required rate of saving, S/Y (the dependent variable), necessary to achieve a desired rate of growth of GDP, $\Delta Y/Y$ (the independent variable), given the coefficient k (the capital/output ratio) and the functional relationship: $S/Y = k(\Delta Y/Y)$. See chapter 4 for a more detailed discussion of the Harrod-Domar model.

THE BASIC TYPES OF PLANNING MODELS

Planning models can conveniently be divided into three basic categories according to the degree of structural complexity and the particular use to which the model is being put. The first and simplest type of planning model is the *aggregate* model which deals with the entire economy in terms of such aggregate components as consumption, production, investment, saving, exports, imports and the like. Aggregate models are usually used to determine possible growth rates of GDP under simplifying assumptions like that of the Harrod-Domar model which assumes that limited capital resources constitute the major constraint on economic development. In countries where inadequate foreign exchange reserves are felt to be the principal bottleneck inhibiting economic growth, the aggregate model might concentrate more on exports, imports, terms of trade fluctuations and sources of foreign financial assistance. In either case, the aggregate model usually provides only a rough first approximation of the general directions which an economy might take. As such, it rarely constitutes the operational development plan. In most instances, the projection of aggregate components of GDP merely provides a general overall framework or initial stage in the formulation of a comprehensive development plan.

The next type of planning model, the so-called *sectoral* model, really comprises two fundamentally different approaches to development programming. The first approach (the one with which we will be dealing in chapter 4), attempts to divide the economy into two or more *main sectors* such as agriculture and non-agriculture, or the consumption-goods sector, the investment-goods sector, and the export sector, etc., with a view towards formulating a complete plan based on the co-ordinated activities of these principal sectors of the economy. We shall call this more detailed approach the *complete main-sector planning* model. A second approach has been to concentrate on levels of production and consumption, not of the entire national economy either as a unique entity or as a composite of a few main sectors, but rather to investigate the possibility of growth in a *single* individual sector. In this type of approach, which we shall call the *single-sector project* model, growth prospects in isolated sectors are assessed on their own merit and specific industrial projects are drawn up on the basis of this partial analysis.

The single-sector project approach is most often undertaken in those economies where statistical data for an aggregate or complete main-sector model are lacking even though detailed information may exist for one or more individual sectors. The main drawback is that the development plan, if based exclusively on a sectoral project approach, often looses its desirable aspects of internal consistency, and more important, overall feasibility. Rather than a well co-ordinated programme of action, the plan could easily emerge as a haphazard collection of assorted development projects with no apparent interconnections. The project approach to planning has been utilized primarily in African-type

economies where the industrial sector is still in its infancy and statistical data is often crude and incomplete. The earlier five-year plans of Ghana (1959–64), Nigeria (1963–68), Kenya (1964–69/70), Tanzania (1964–69/70) and Uganda (1961/62–65/66), could be described generally as sectoral project plans although considerable effort was exerted by the Planning Ministries of these countries to co-ordinate as best they could the individual investment projects in the light of the limited statistical information available.

The third and most sophisticated approach to planning is the *interindustry* approach in which the activities of all productive sectors of the economy are interrelated with one another in the context of a set of simultaneous linear equations expressing the specific production processes of each industry. Direct and indirect repercussions of exogenous changes in the demand for the products of any one sector on output, employment, and imports of all other sectors are traced throughout the entire economy in an intricate web of economic interdependences. Given the planned output targets for each sector of the economy, the interindustry model can be used to determine intermediate material, import, labour, and capital requirements with the result that a comprehensive economic plan with mutually consistent production levels can be constructed. Interindustry models range from simple input-output models, usually consisting of from ten to thirty sectors in the developing economies and from thirty to four hundred sectors in advanced economies, to the more complicated linear-programming activity analysis models where checks of feasibility and optimality are built into the model in addition to the criterion of internal consistency that is the distinguishing feature of the input-output approach.

Interindustry models, especially of the programming variety, are primarily applicable in economies that have achieved a minimum degree of industrial development as characterized by a significant volume of interindustry transactions. However, simple input-output models have been put to a number of useful tasks in those developing economies which have begun to move along the path of industrialization.

THE DIVISION OF THE BOOK

Having completed these introductory remarks on the various aspects of economic planning, we can now proceed to analyse the role of planning both in the advanced collectivist economies of the Soviet variety and the developing 'mixed' economies of the African distinction. We shall approach the question of planning from the context of economic models since it is our belief that the 'thought processes' of central collectivist planning as well as the actual procedure of development planning can be explained best with the aid of certain theoretical economic models. We shall begin by examining the nature of economic planning in a mature, fully developed, centralized socialist economy where the government actively controls the specific movements of all economic variables. For

this endeavour we shall make use of a simplified input-output interindustry model which is constructed in chapter 2. In chapter 3, a hypothetical central planning problem is posed and solved using the input-output techniques which were elucidated in the previous chapter. In the latter part of chapter 3 we will attempt to illustrate the close relationship between the interindustry model and central planning with the aid of a brief survey of the central planning process of the Soviet economy and the important role of the 'material balance' in the Soviet plan. In chapters 4 and 5 we turn to the question of development planning and the role of various theoretical and quantitative economic models in formulating development plans, especially in the African context. Chapter 4 analyses aggregate and main-sector models while chapter 5 discusses various types of interindustry activity-analysis models.

Suggested Readings

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Sukhamoy Chakravarty, *Capital and Development Planning* (Cambridge, Mass., MIT Press, 1969).
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CHAPTER 2

Input-Output Analysis:

THE BASIC FRAMEWORK

We shall begin our discussion of planning by examining the operational mechanism of a complete centrally planned economy which does not, or at least need not, make use of price movements to allocate resources. 'Accounting' prices exist, but they are established more or less arbitrarily by the central planning unit, and they are fixed over long periods of time. Adjustment to shifts in tastes and technology occur through *planned* shifts in production rather than through price changes which in turn elicit the production shifts which bring about the necessary reallocation of resources in market economies.

If economic decisions are to be made centrally, there must be some means of centralizing the information necessary to make proper decisions. One of the best 'tools' for laying out the main information desired for centrally planned economies is the input-output table. Input-output analysis may be used implicitly or explicitly, but the 'thought processes' involved in central planning are the 'thought processes' involved in input-output analysis.

And so our first task is to understand an input-output table and some of the rudimentary mathematics of input-output models. In chapter 3 we will work through a hypothetical planning problem and try to show how an input-output framework can be used to describe and elucidate the 'method of material balances' employed for central planning in the Soviet Union, as well as other, 'looser' planning operations such as have been employed in certain Western countries.

THE STRUCTURAL NATURE OF AN INPUT-OUTPUT TABLE

WHAT IS INPUT-OUTPUT?

An input-output table can be regarded as a collection of data describing the particular structural characteristics of any economic system, and/or as an analytic technique for explaining and influencing the behaviour of the system at a certain

point in time or over the course of time.[1] The basic notion of input-output analysis rests on the belief that the economy of any country can be divided into a distinct number of sectors, called industries (or sometimes 'activities'), each consisting of one or more firms producing a similar but not necessarily homogeneous product. Each industry requires certain *inputs* from other sectors in order to produce its own *output*. Similarly, each industry sells some of its gross output to other industries so that they too can satisfy their intermediate material needs. The input-output table provides a convenient framework for measuring and tracing these interindustry flows of current inputs and outputs among the various sectors of the economy.

THE MAKE-UP OF A TYPICAL TABLE

In Table 2.1, we have a representation of the structural make-up of a typical input-output table. Most of the important information contained in the table is located within the three main quadrants; the 'Interindustry Transactions' quadrant (II), the 'Value Added' quadrant (III), and the 'Final Use' quadrant (I). The 'Direct Factor Purchase' quadrant (IV) is not as important for planning purposes as the other three although it is necessary for accounting purposes, especially for measuring gross domestic product.

The producing sectors of the economy are listed as rows 1, 2, . . . n, in the transactions quadrant while these same industries are also listed by column as using sectors. The transactions quadrant is thus always a 'square' matrix

[1] The historical origin of the basic idea of input-output analysis can be traced back to François Quesnay and his famous *Tableau Economique* first presented in 1758. The *Tableau* was a diagram designed to show how goods and services were circulated among the four socio-economic classes of that period; the land owners, the farmers, the manufacturers, and the traders. It also provided a method of showing how the income of each class was related to the interdependent activities of all four classes in both production and consumption. As such, Quesnay's *Tableau* was the first representation of the general interdependence of all sectors and all elements of the economic process.

While Quesnay's *Tableau* marked the origin of the recognition of the fundamental interdependence of all economic activities which is the core of input-output analysis, the real inspiration for modern work in this field comes from the endeavours of Leon Walras. The Walrasian general equilibrium system attempts to demonstrate the mathematical nature of the interdependence between producing and consuming sectors of the economy in the context of a set of simultaneous linear equations. It shows how activities of different sectors of the economy will be affected by the slightest change in any of the important independent variables. Thus, Walras carried Quesnay's conception of economic interdependence considerably further by providing an appropriate mathematical framework. As we know, however, the Walrasian general equilibrium model is based on the highly restrictive assumption of perfect competition in all product and factor markets, and as such, can hardly be used as a practical tool of analysis in contemporary economies. Therefore, it was not until Wassily Leontief devised his input-output tables for the American economy in the 1930s that inter-industry analysis really became a tool of *applied* economics that could be used to analyse real world economic phenomena.

TABLE 2.1

THE MAKE-UP OF A TYPICAL INPUT-OUTPUT TABLE

Using Sectors (Inputs) ↓ / *Producing Sectors (Outputs)* →	*Intermediate Use Columns* (1) Sector	(2) Sector	.... N Sector	Total Intermediate Production	*Final Use (Demand) Columns* Private Consumption Investment; Government Consumption Investment; Exports	*Total Final Demand*	*Total Output (Sales)*
1. Sector 1							
2. Sector 2 . .			Outputs Produced for Intermediate Use	+	Outputs Produced for Final Use	=	*Total Output Sector 2*
Rows . . . . N. Sector N		Inputs of Materials Purchased	**Interindustry Transactions Quadrant (II)**		**Final Use Quadrant (I)**		. . . . . . Total Output of Sector N
Total Intermediate Use		+					
Rows Labour, Capital, Land, Natural resources		Primary Inputs Purchased and Used	**Value Added Quadrant (Primary Inputs) (III)**		**Direct Factor Purchase Quadrant (IV)**		
Value Added		=					
Total Inputs (Purchases)		Total Inputs Sector 2					*Gross Output*

Total Output of Sector i=Total Input of Sector *i*, where i=1, 2, . . . n.

with the same number of rows as columns, one for each sector of the economy.[2] Reading from left to right along any row, say row 2, which represents sector or industry 2, we see that part of the total *output* of sector 2 is sold to the other sectors for *intermediate use* in the production of their own output (e.g. cotton growers sell their output to the textile industry who use this raw material input to produce cotton fabrics). Reading down any column in the transactions quadrant, we note that each particular sector (sector 2) also *uses* the outputs of other industries as material *inputs* for its own production (e.g. cotton growers purchase fertilizer and chemicals to increase and protect their per-acre cotton yields). In short, *rows* designate *outputs, columns* represent *inputs*, and each sector is both a user of inputs and a producer of outputs.

Turning to the final use quadrant, we see that the outputs of each sector are also normally demanded for ultimate use in the form in which they are produced, e.g. as final demand products which are consumed for their own sake and not for use in further production.[3] They may be purchased as consumption goods by individual consumers or by the central government; as investment goods by the state or by private investors if we are dealing with a mixed market economy; or, they may be sold to foreign demanders in the form of exports.[4] The horizontal summation of the outputs of sector 2 sold as intermediate goods to the other sectors plus the output sold to private, public and foreign final demanders yields the *total output* of sector 2. This figure is listed as the second element in the last column of the table. Similarly, the total output of the other n–1 sectors can also be arrived at in the same manner.

Reading down columns 1 to n in the value added quadrant we obtain information about the amount of *primary inputs* (i.e. land, labour, capital,[5] etc.) used by each sector in the production of its output. The sum of the value of these primary inputs used in the whole economy yields the total value added by the n industries. For any particular sector the elements in each column of this quad-

[2] A 'matrix' is a rectangular array of numbers. The size of the matrix is indicated by the number of rows (m) and columns (n), written as m x n. In a square matrix, there are an equal number of rows and columns. Thus, a square matrix that consisted of 5 rows and 5 columns would be represented as a 5×5 matrix.

[3] Sugar can be purchased for final use by the individual household (e.g. to sweeten a cup of tea) or it might be bought by the soft drink industry for intermediate use in the production of soft drinks.

[4] The sum total of final demands for the products of the n sectors minus the value of intermediate imports plus any direct factor purchases by final demanders would be representative of gross domestic product.

[5] The distinction between the primary input row 'capital' and the final demand column(s) 'investment' is merely our familiar distinction between a *stock* and a *flow*. The capital row shows that part of the existing stock of capital that is used up (e.g. due to depreciation) during the year while the investment column represents the flow of new investments during the year, i.e. the change in the total capital stock.

rant when added to the elements in the corresponding column of the transactions quadrant immediately above yield a value for *total inputs purchased or used up* by this industry during the accounting period, i.e. the total costs of operation. The first n elements of the bottom row of the input-output table give cost figures for each sector. Thus, total payments for materials and factors by sector 2 would be obtained by reading the second element from this last row. For each sector, the value of total output must equal the value of total input, i.e. the horizontal summation of the elements in row 1 must be equal to the vertical summation of elements in column 1, etc.

Finally, the direct factor purchase quadrant shows those primary inputs which are employed by final users. The main component of this quadrant would be the purchase of labour services by the central government. For example, in most African economies government employment in the various ministries and branches of the civil service is ordinarily quite substantial. Total government expenditure for labour services would therefore be entered in this quadrant in the box where the labour row intersects the government 'consumption' column. Households also purchase labour in the form of domestic help and the value of such activities would be entered accordingly at the intersection of the labour row and private 'consumption' column. The sum of all entries in quadrants III and IV yields an alternative figure for gross domestic product.[6]

A HYPOTHETICAL EXAMPLE

Let us turn now to an input-output table using hypothetical data and review the above procedures with the aid of numerical figures. Table 2.2 provides such a numerical illustration of the basic input-output accounting system.[7] For simplicity, we have divided our table into only 5 sectors (agriculture, sector 1; extractive industry, sector 2; manufacturing, sector 3; power, sector 4; and transportation, sector 5). Actual tables in use range in size from 20 to 400 sectors.

Note that each sector appears in the accounting system twice, as a producer of outputs (rows 1 to 5) and as a user of inputs (columns 1 to 5). The elements in each row show how a particular sector disposed of its total output during the given accounting period. For example, of the total available output of agricultural products (125 units),[8] 15 units are used by agriculture itself, 20 by

[6] For a much more detailed discussion of input-output accounting techniques, see B. Van Arkadie and C. Frank, *Economic Accounting and Development Planning* (Nairobi, Oxford University Press, 1966), chapter VIII.

[7] In actual tables each 'unit' of output would ordinarily be valued at *constant prices* in the prevailing currency. For example, in Table 2.2, 125 units of agricultural output might represent £125,000. As long as all output is valued at constant prices, no great problem arises when prices change.

[8] For the present, disregard all symbols in parentheses and concentrate only on the numerical values.

Table 2.2
HYPOTHETICAL INPUT-OUTPUT TABLE

Using Sector (Inputs) / *Producing Sectors (Outputs)*	*Intermediate Use*						*Final Use (Demand)*					
	Agriculture	Extractive Industry	Manufacturing	Power	Transportation	*Total Intermediate Demand*	Household Consumption	Investment	Non-Investment Expenditure	Exports	*Total Final Demand*	*Total Output*
Agriculture	15(x_{11})	0(x_{12})	20(x_{13})	0(x_{14})	10(x_{15})	45 $\sum_{j=1}^{5} x_{1j}$	35	10	5	30	80(Y_1)	125(X_1)
Extractive Industry	0(x_{21})	0(x_{22})	0(x_{23})	0(x_{24})	0(x_{25})	0 $\sum_{j=1}^{5} x_{2j}$	0	10	0	30	40(Y_2)	40(X_2)
Manufacturing	10(x_{31})	0(x_{32})	25(x_{33})	15(x_{34})	5(x_{35})	55 $\sum_{j=1}^{5} x_{3j}$	15	20	5	5	45(Y_3)	100(X_3)
Power	5(x_{41})	15(x_{42})	15(x_{43})	0(x_{44})	15(x_{45})	50 $\sum_{j=1}^{5} x_{4j}$	5	10	10	0	25(Y_4)	75(X_4)
Transportation	5(x_{51})	10(x_{52})	15(x_{53})	0(x_{54})	5(x_{55})	35 $\sum_{j=1}^{5} x_{5j}$	5	8	2	0	15(Y_5)	50(X_5)
Total Purchases	35	25	75	15	35	185	60	52	22	65	205	
Imports	15(m_1)	0(m_2)	10(m_3)	30(m_4)	5(m_5)	60(M_T)	5	5	0	0	(10)	60(70)
Government (Taxes)	20	5	3	7	2	37	(35)	(0)	(0)	(20)	(55)	37(92)
Households (Labour)	40(L_1)	5(L_2)	6(L_3)	5(L_4)	2(L_5)	58(L_T)	1	0	12	0	13	71
Capital (C)	5(C_1)	3(C_2)	5(C_3)	12(C_4)	4(C_5)	29(C_T)	0	0	0	0	0	29
Natural Resources (N)	10(N_1)	2(N_2)	1(N_3)	6(N_4)	2(N_5)	21(N_T)	0	0	0	0	0	21
Value Added	75	15	15	30	10	145	1	0	12	0	13	158
Total Inputs	125	40	100	75	50	390	66	63	34	65	218	608

manufacturing, and 10 by transportation. The total intermediate use of agricultural products, i.e. use for further production, is 45 units. To this figure must be added the quantity of agricultural goods demanded by final users which in our table consists of the consumption of 35 units by households, total government expenditure on 15 units and exports to foreign countries of 30 units. The sum of total intermediate and total final demand yields a gross output for agricultural production of 125 units. Similarly, we see that the extractive sector produces a total output of 40 units of which none is sold on an interindustry basis to other sectors while the government purchases 10 units and 30 units are exported. The disposition of the total outputs of the other three sectors can be read off the table in the same manner.

The role of the agricultural sector as a purchaser of *inputs* is shown by *column one.* Reading down this column we see that in order to produce its total output of 125 units, agriculture had to use 15 units of its own output (e.g. using a portion of sisal or coffee output for replanting), 10 units of manufacturing output (e.g. fertilizers, insecticides, etc.), 5 units of power (e.g. to operate rotating water sprays and other electrical equipment), and 5 units of transportation's product (e.g. for transporting perishable goods to local markets or to the coast for export). Thus, the total domestic interindustry purchases of intermediate material goods and services by agriculture was 35 units. The remaining 90 units of total inputs purchased consisted of the importation of 15 units of foreign goods and the creation of 75 units of *value added* in the form of payments of 20 units to the government as taxes, 40 units to households as wages, 5 units for the use of capital, and 10 units for the use of land.[9] It is immediately evident that the value of the total output of agriculture is equal to the total value of all inputs purchased, i.e. 125 units. This same procedure can be followed in analysing the input-output structure of each and every sector of the economy.

In terms of national income accounting techniques, we can arrive at identical figures for gross domestic product by using either the income or the product approach. Using the income or value added approach, GDP is determined as the summation of total payments (both intermediate and final) to primary inputs plus business tax payments.[10] In Table 2.2, we see that these total payments amounted to 158 units. Using the product approach which defines GDP as the difference between *total* final demand (80+40+45+25+15+13= 218) and intermediate imports (60) we arrive at the same figure of 158 units

[9] In a collectivist economy these last two components of value added would still be calculated for accounting purposes although no actual payment need be made.

[10] Payments of taxes by households (e.g. income and poll taxes) and by exporters (e.g. export taxes and licence fees), which amounted to 35 and 20 units respectively, do not get included in GDP calculations. To indicate their non-inclusion we have put these items in parentheses. They are important for assessments of total government revenue, however, and therefore should not be omitted from the table.

for gross domestic product. The input-output table can also be used to show the sources (row 8) and uses (columns 8 and 9) of government funds,[11] or the current account of the balance of payments, e.g. exports (column 10) minus imports (row 7) would yield a net 'deficit' of 5 units (i.e. 65–70=–5) from our hypothetical table.[12]

So far, we have been concerned primarily with a descriptive explication of the input-output table. Even if input-output were used only as an accounting mechanism, its highly interdependent, disaggregated contents would still represent a descriptive tool of considerable analytic value. However, the major theoretical and practical value of input-output tables is that they can easily be transformed into a consistent mathematical model and utilized either as a forecasting tool to predict the effects of autonomous changes in final demand on *total* output and employment in all sectors of the economy, or, as in centralized socialist economies, as a framework within which consistent, comprehensive economic plans can be drawn up and carried out. In order to transform the accounting table into a workable mathematical model, however, certain necessary assumptions about the production process must be made.

THE VITAL ASSUMPTION: CONSTANCY OF INPUT COEFFICIENTS

The crucial assumption of input-output analysis, i.e. the one that makes the system operationally effective, is the assumption that a *single process production function* exists in every industry. Actually, this characteristic assumption of input-output can be broken down into two closely related but distinct constituent parts. The first is the assumption of *constant returns to scale.* The second, and by far the more controversial, is the corresponding assumption that no *substitution among inputs* is possible in the production of any good or service. An alternative way of stating this second assumption is that since there is only one process or method of production in each industry, the level of output of a product determines uniquely the level of each input required. Technically we may say that the production process is assumed to be characterized by the existence of *constant* 'technical coefficients of production', i.e. each *additional* unit of new output is produced by an unchanging proportional

[11] Note that the total receipts of government funds (i.e. total revenue) are given by the figure in parenthesis (92) at the far end of row 8. (See preceeding footnote for explanation.)

[12] The figure for total imports, 70, is put in parenthesis for accounting purposes once again. The 10 units of final demand imports have already been recorded within the final use quadrant. Thus, the five units of imports purchased by consumers must have come from one of the five sectors of the economy. We would be guilty of 'double counting' if these final demand imports were included in our calculations of GDP. However, if they were not distinguished separately our balance of payments accounts would be erroneous—thus, the parenthesis.

combination of material inputs from the other sectors. For example, in Table 2.2 we see that in order to produce its 100 units of output, the manufacturing sector had to purchase 20 units of agricultural output for its intermediate input needs. Dividing 20 by 100 we find that our proportionality assumption indicates that for every unit of manufacturing output, 0.20 units of agricultural products will always be required as inputs so long as the production process of the manufacturing sector remains unchanged by the prevailing technology. Similarly, for every unit of its output produced, the manufacturing sector requires 0.25 units of its own goods (25/100= 0.25), 0.15 units of power (15/100=0.15), and 0.15 units of transportation (15/100= 0.15) as necessary material inputs. These technical coefficients of production for manufacturing were obtained by dividing each element in column 3 of the transactions matrix by the total output of the manufacturing sector, 100. More formally, by letting the symbol a_{ij} represent the number of units of the ith product, say manufacturing (where $i=1, 2, \dots, n$ and designates the *row* in which the industry is located), necessary to produce *one unit* of output of the jth sector, say transportation (where j also equals any number from 1 to n and each particular number designates the *column* location of that particular industry within the transactions matrix), we can derive a matrix of technical coefficients of production with the same number of rows, columns and elements as contained in the transaction matrix of Table 2.2, i.e. 5 rows, 5 columns, and 25 elements. For example, a_{13} would designate the coefficient located in the 1st row and 3rd column of the derived technical matrix. It would represent the number of units of the 1st product (agriculture) required by the 3rd sector (manufacturing) in order to produce a unit of its own (i.e. manufacturing) output. As was noted above, a_{13} in our example would be equal to 20/100 or 0.20. It is immediately evident that the calculation of all a_{ij}'s is a relatively straightforward task. We simply divide the number located in the ith row and jth column of the original transactions matrix by the total output of industry j. Each column of the new matrix comprises the input coefficients of one particular sector, j. It represents the single-process *production function* of that industry. Thus, the entire 'A' matrix, as the matrix of technical coefficients is often designated, summarizes the production processes of the entire economy in the form of goods that flow into and out of each industry.

(2.1) Stated mathematically, $a_{ij}= \dfrac{x_{ij}}{X} \quad \begin{matrix} i=1, 2, \dots, n \\ j=1, 2, \dots, n \end{matrix}$

where,

x_{ij} represents the actual number of units of good i used by industry j as indicated by the figure in the ith row and jth column of the transaction matrix, and

X_j equals the *total output* of industry j as indicated by the last figure in the jth row of the input-output table.

The matrix of technical coefficients of production for any input-output table with n sectors would consist of n x n elements. For a table with only 5 sectors, as in our example, the 25 technical coefficients of the matrix would be arranged symbolically as follows:

Table 2.3

A 5×5 Matrix of Technical Coefficients

	Sector 1	*Sector 2*	*Sector 3*	*Sector 4*	*Sector 5*
Sector 1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
Sector 2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
Sector 3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
Sector 4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
Sector 5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Using equation (2.1) above to calculate the a_{ij}'s for our hypothetical 5 sector input-output table, we can arrive at the following 'A' matrix of technical coefficients of production:

Table 2.4

The 'A' Matrix of Technical Coefficients for our Hypothetical 5 Sector Economy

	Agriculture	*Extractive Industry*	*Manufac-turing*	*Power*	*Transpor-tation*
Agricul-ture	15/125=*.12*	0/40=*0*	20/100=*.20*	0/75=*0*	10/50=*.20*
Extrac-tive indu-stry	0/125=*0*	0/40=*0*	0/100=*0*	0/75=*0*	0/50=*0*
Manufac-turing	10/125=*.08*	0/40=*0*	25/100=*.25*	15/75=*.20*	5/50=*.10*
Power	5/125=*.04*	15/40=*.375*	15/100=*.15*	0/75=*0*	15/50=*.30*
Transpor-tation	5/125=*.04*	10/40=*.25*	15/100=*.15*	0/75=*0*	5/50=*.10*

Thus, for example, we see that the amount of manufactured goods required to produce a unit of agricultural output, a_{31}, is 0.08; the amount of power necessary to produce a unit of output of the extractive industry, a_{42}, is 0.375; the amount of transportation needed to produce a unit of power output, a_{54}, is 0; and the necessary agricultural *input* required to produce a unit of agricultural *output*, a_{11}, is 0.12. Since these technical coefficients of production are assumed to be *constant* over time, the input-output table can be utilized to measure the *direct* and the *indirect* effects on the entire economy of any sectoral change in total output or final demand. But in order to understand more fully how this process actually works, we must now turn to see how the input-output table can be transformed into an *operational* mathematical model.

THE MATHEMATICAL INPUT-OUTPUT MODEL

THE DUAL FOUNDATION OF THE BASIC MODEL

The analytical and mathematical content of the input-output model rests on a dual foundation. The first consists of a set of *accounting* equations, one for each producing sector of the economy. The first of these equations states that the total output of sector 1 is equal to the sum of the separate amounts sold by sector 1 to the other industries plus the amount produced to satisfy final demands. The second equation says the same thing for sector 2—and so on for all n industries. In terms of our input-output table these equations state that for any sector total output is equal to the sum of all the entries in that sector's row in the table. Thus, an implicit assumption of input-output analysis common to all general equilibrium models is that in all sectors the entire product produced is consumed either by other industries as intermediate inputs or by final demanders. In short, supply always equals demand. We can state this first equation in concise symbolic terms as follows:[13]

Let,

X_i measure the annual rate of total output (in the appropriate value units) of industry i,

x_{ij} represent the amount of the product of industry i absorbed annually as an intermediate input by industry j,

and finally, let

Y_i equal the amount of the same product i produced to satisfy 'final demand'.

The overall input-output accounting balance for the entire economy comprising n separate industries or sectors can now be described in terms of n linear equations:

$$(2.2)\quad \sum_{j=1}^{n} x_{ij} + Y_i = X_i$$

where $i=1, 2, \ldots, n$.

Each equation is representative of the fact mentioned earlier that in all sectors the entire product produced (X_i) is consumed either by the other industries $\left(\sum_{j=1}^{n} x_{ij}\right)$ or by final demanders (Y_i). For example, in Table 2.2 we would have five such accounting equations, the one for industry 3,

[13] To facilitate comprehension of the precise relationships involved in the following model in the context of our hypothetical input-output table, the student should refer back to the symbols contained in the parentheses in Table 2.2 to see just how these same symbols that will be employed in our model relate to the particular numerical entries in the table.

manufacturing, being of the following order:

$$\overbrace{x_{31} + x_{32} + x_{33} + x_{34} + x_{35}}^{\sum_{j=1}^{5} x_{3j}} + Y_3 = X_3$$

or, substituting the appropriate numerical figures,

$$10+0+25+15+5+45=100.$$

The second and more important foundation of the input-output model consists of another set of n equations, one for each industry, describing the input-output structure of that industry in terms of a derived set of a_{ij} technical coefficients of production. Thus, the commodity flows, x_{ij}, included in the above balance equations are subject to the following set of structural relationships:

(2.3) $x_{ij}=a_{ij}X_j$[14] $\quad i=1, 2, \ldots, n.$

$\qquad\qquad\qquad j=1, 2, \ldots, n.$

Substituting for x_{ij} from (2.3) into (2.2) and transposing terms, we obtain the following expression:

(2.4) $X_i - \sum_{j=1}^{n} a_{ij}X_j = Y_i$

where $i=1, 2, \ldots, n.$

In terms of our hypothetical economy, system (2.4) would consist of 5 linear equations that would be written symbolically and numerically as follows:

Symbolic Representation

$X_1-a_{11}X_1-a_{12}X_2-a_{13}X_3-a_{14}X_4-a_{15}X_5=Y_1$

$X_2-a_{21}X_1-a_{22}X_2-a_{23}X_3-a_{24}X_4-a_{25}X_5=Y_2$

$X_3-a_{31}X_1-a_{32}X_2-a_{33}X_3-a_{34}X_4-a_{35}X_5=Y_3$

$X_4-a_{41}X_1-a_{42}X_2-a_{43}X_3-a_{44}X_4-a_{45}X_5=Y_4$

$X_5-a_{51}X_1-a_{52}X_2-a_{53}X_3-a_{54}X_4-a_{55}X_5=Y_5$

Numerical Representation

125–.12 (125)–0 (40)–.20 (100)–0 (75)–.20 (50)=80

40–0 (125)–0 (40)–0 (100)–0 (75)–0 (50)=40

100–.08 (125)–0 (40)–.25 (100)–.20 (75)–.10 (50)=45

75–.04 (125)–.375 (40)–.15 (100)–0 (75)–.30 (50)=25

50–.04 (125)–.25 (40)–.15 (100)–0 (75)–.10 (50)=15

[14] Since a_{ij} has already been defined as being equal to x_{ij}/X_j, system (2.3) describing the structural relationship is merely another way of expressing this definition of proportionality, i.e. $a_{ij} = \frac{x_{ij}}{X_j} \Rightarrow x_{ij} = a_{ij} X_j$.

For convenience let us now rewrite system (2.4) in terms of matrix and vector notations as follows:

(2.5) $\bar{X}-[A].\bar{X}=\bar{Y}$

where,

$\bar{X}$ represents a *column vector* [15] of total outputs consisting of n elements, in our example n=5, each of which numerically represents the total output of one of the n industries;

[A] is a n x n square matrix of technical coefficients of production described earlier; and,

$\bar{Y}$ is a column vector of total final demands.

Thus, if the economy were divided up into 30 sectors (i.e. n=30), system (2.5) would be a convenient way of avoiding the tedious task of writing out a set of 30 simultaneous linear equations.

A BRIEF REVIEW OF MATRIX ALGEBRA

It might prove helpful at this point to review some fundamental techniques of matrix algebra which will be used extensively in the following pages to demonstrate the mathematics of input-output analysis. Similarly, we will try to explain further concepts of matrix algebra whenever the discussion seems to warrant such an explanation. Those already familiar with matrix algebra need not bother with these sections. However, it behoves the unfamiliar to study them carefully. They are not difficult but they do require close attention.

Matrix *addition* or *subtraction* consists of adding or subtracting corresponding elements, i.e. those having the same subscripts i and j. This can only be done if the two (or more) matrices have the *same* number of rows and columns. For example, two 2×2 matrices would be added or subtracted as follows:

$$[A] \pm [B] = \begin{Bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Bmatrix} \pm \begin{Bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Bmatrix} = \begin{Bmatrix} (a_{11} \pm b_{11}) & (a_{12} \pm b_{12}) \\ (a_{21} \pm b_{21}) & (a_{22} \pm b_{22}) \end{Bmatrix}.$$

[15] A 'vector' is an ordered array of elements assembled either vertically ('column' vector) or horizontally ('row' vector). A column vector is really a special case of an n x 1 matrix, i.e. a matrix that has n rows and 1 column. A row vector is merely a 1 x n matrix, i.e. 1 row and n columns. In our example, the column vector of total output would be written as follows:

$$\bar{X} = \begin{vmatrix} 125 \\ 40 \\ 100 \\ 75 \\ 50 \end{vmatrix} . \text{ Similarly, } \bar{Y} = \begin{vmatrix} 80 \\ 40 \\ 45 \\ 25 \\ 15 \end{vmatrix} \text{ is a column vector of final demands.}$$

Matrix *multiplication* consists of multiplying each element of a row (i) in the first matrix by the corresponding element in a column (j) of the second matrix. The resultant products are then added together to give the element located in the ith row and jth column in the product matrix. For example,

$$\text{if } [A] = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \text{ and } [B] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

then [A] × [B] would be calculated as:

$$\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (2.1+4.3) & (2.2+4.4) \\ (5.1+6.3) & (5.2+6.4) \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 23 & 34 \end{bmatrix}$$

Matrix multiplication can only be carried out if the number of columns in the first matrix is equal to the number of rows in the second. Thus, the product of an m x n matrix and an n x p matrix, is an m x p matrix, whereas an m x n matrix could not be multiplied by an m x p matrix since the number of columns, n, in the first matrix would not equal the number of rows, m, in the second matrix.

In input-output analysis, we usually have to multiply a matrix by a column vector as in system (2.5). By the above rule, this can only be accomplished if the column vector is the second term. In situations of this sort we must *premultiply* the vector by the matrix so that the elements in each row of the matrix are multiplied by the corresponding elements of the column vector. The result is also a column vector, since multiplying n x n by n x 1 gives a product matrix which is n x 1. For example, in system (2.5) the expression $[A].\bar{X}$ represents the product of the matrix of technical coefficients and the column vector of total outputs and yields a column vector of total intermediate demands $\sum_{j=1}^{n} x_{ij}$. The expression may be written as follows:

$$[A]\cdot\bar{X} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} \cdot \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{vmatrix} = \begin{vmatrix} a_{11}X_1+a_{12}X_2+a_{13}X_3+a_{14}X_4+a_{15}X_5 \\ a_{21}X_1+a_{22}X_2+a_{23}X_3+a_{24}X_4+a_{25}X_5 \\ a_{31}X_1+a_{32}X_2+a_{33}X_3+a_{34}X_4+a_{35}X_5 \\ a_{41}X_1+a_{42}X_2+a_{43}X_3+a_{44}X_4+a_{45}X_5 \\ a_{51}X_1+a_{52}X_2+a_{53}X_3+a_{54}X_4+a_{55}X_5 \end{vmatrix} = \begin{vmatrix} \sum_{j=1}^{5} x_{1j} \\ \sum_{j=1}^{5} x_{2j} \\ \sum_{j=1}^{5} x_{3j} \\ \sum_{j=1}^{5} x_{4j} \\ \sum_{j=1}^{5} x_{5j} \end{vmatrix}.$$

If this column vector of intermediate demands is then subtracted from the column vector of total outputs, we obtain the column vector of final demands.

Thus,

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{vmatrix} - \begin{vmatrix} \sum_{j=1}^{5} x_{1j} \\ \sum_{j=1}^{5} x_{2j} \\ \sum_{j=1}^{5} x_{3j} \\ \sum_{j=1}^{5} x_{4j} \\ \sum_{j=1}^{5} x_{5j} \end{vmatrix} = \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{vmatrix}$$

which according to our rule for matrix (and thus vector) subtraction brings us back to our 5 'accounting' equations of system (2.2) namely:

$$X_i - \sum_{j=1}^{5} x_{ij} = Y_i \quad i=1, 2, \ldots, 5.$$

THE LEONTIEF MATRIX

We can now *premultiply* both sides of system (2.5) by the 'identity' or 'unit' matrix, denoted [I], to obtain the following expression:

(2.6) $[I] . \bar{X} - [A] . \bar{X} = \bar{Y}$ [16]

[16] The 'identity' matrix is merely a square matrix in which all the diagonal elements reading from left to right have a value of 1 while all other elements are equal to 0. Thus, a 3×3 identity matrix would be:

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The operation of the identity matrix in matrix algebra has the same effect as the operation of the number 1 in simple algebraic equations, i.e. multiplication leaves the value of all elements in the system unchanged.

Thus, if a column vector (e.g.$\bar{X}$) is pre-multiplied by an identity matrix [I], we obtain the same column vector with unchanged values. For example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{vmatrix} 2 \\ 1 \\ 4 \end{vmatrix} = \begin{vmatrix} (1.2) + (0.1) + (0.4) \\ (0.2) + (1.1) + (0.4) \\ (0.2) + (0.1) + (1.4) \end{vmatrix} = \begin{vmatrix} 2 \\ 1 \\ 4 \end{vmatrix}$$

or,

symbolically $[I].\bar{X} = \bar{X}$.

Factoring out the $\bar{X}$ column vectors on the left side of system (2.6), we derive a very familiar expression of input-output mathematics, namely:

(2.7) $(I-A) . \bar{X} = \bar{Y}$.

The (I—A) matrix, often called the 'Leontief matrix' in honour of the 'father' of input-output analysis, is obtained mathematically by subtracting each element of the 'A' matrix from its counterpart in the 'I' matrix.

Thus, in our example *symbolically*

$$(I-A) = \begin{bmatrix} 1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1 \end{bmatrix} - \begin{bmatrix} a_{11}&a_{12}&a_{13}&a_{14}&a_{15}\\a_{21}&a_{22}&a_{23}&a_{24}&a_{25}\\a_{31}&a_{32}&a_{33}&a_{34}&a_{35}\\a_{41}&a_{42}&a_{43}&a_{44}&a_{45}\\a_{51}&a_{52}&a_{53}&a_{54}&a_{55} \end{bmatrix}$$

$$= \begin{bmatrix} (1-a_{11})&-a_{12}&-a_{13}&-a_{14}&-a_{15}\\-a_{21}&(1-a_{22})&-a_{23}&-a_{24}&-a_{25}\\-a_{31}&-a_{32}&(1-a_{33})&-a_{34}&-a_{35}\\-a_{41}&-a_{42}&-a_{43}&(1-a_{44})&-a_{45}\\-a_{51}&-a_{52}&-a_{53}&-a_{54}&(1-a_{55}) \end{bmatrix}$$

or *numerically*

$$(I-A) = \begin{bmatrix} 1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1 \end{bmatrix} - \begin{bmatrix} .12&0&.20&0&.20\\0&0&0&0&0\\.08&0&.25&.20&.10\\.04&.375&.15&0&.30\\.04&.25&.15&0&.10 \end{bmatrix}$$

$$= \begin{bmatrix} .88&0&-.20&0&-.20\\0&1.0&0&0&0\\-.08&0&.75&-.20&-.10\\-.04&-.375&-.15&1.0&-.30\\-.04&-.25&-.15&0&.90 \end{bmatrix}.$$

Since no coefficient in the 'A' matrix can be greater than one, all elements except those on the diagonal of the (I—A) matrix must be a number whose value is either zero or falls between zero and minus one.[17] All diagonal elements will be positive and range from zero to one (as in the example given above).

[17] An $a_{ij} > 1$ would imply that for each £1 worth of sales of industry j, it would have to purchase more than £1's worth of inputs from industry i. Not only would this be irrational from the business point of view but it would also violate the input-output criterion that for every sector the value of total output must equal the value of total inputs.

INVERTING THE LEONTIEF MATRIX

We come now to the crucial mathematical manipulation of input-output analysis—the one that gives the system its predictive and planning potentialities. Given a matrix of a_{ij} technical coefficients of production assumed to be constant and independent of the volume of output, and the column vector of final demands ($Y_1, Y_2, \ldots, Y_n$), we are now able to solve system (2.7) for all values of total output ($X_1, X_2, \ldots, X_n$) merely by dividing both sides of system (2.7) by the Leontief matrix (I–A) to obtain the following expression:

$$\bar{X} = \frac{\bar{Y}}{(I-A)}$$

(2.8) or $\bar{X} = (I-A)^{-1}\,\bar{Y}$.[18]

The expression $(1\text{–}A)^{-1}$ is called the 'inverted' Leontief matrix and is commonly designated by the block letter 'R', i.e. system (2.8) may be written as:

(2.8a) $\bar{X} = [R]\,.\,\bar{Y}$.

Thus, the matrix operation corresponding to finding $1/a$ in the equation $x = (\frac{1}{a})y$ is called *matrix inversion*, and the result is the reciprocal or inverse matrix. The inverse of matrix [A] is denoted $[A]^{-1}$. The inverse of [A] (or [I−A]) is defined as that matrix which when muliplied by A (or [I−A]) gives the identity matrix I. Therefore, $[A]\,.\,[A]^{-1} = [I] = [1-A]\,.\,[I-A]^{-1}$, etc. When multiplying a matrix by its inverse the order of multiplication does not matter. Note that the inverse is defined only for square matrices. For example, let us calculate the inverse of a 2×2 Leontief matrix. The procedure is quite involved mathematically since the computation of an inverse for any square matrix larger than 3×3 would require n^n computations where n equals the number of rows (or columns). Thus, the inverse of a 4×4 matrix would involve 4^4 or 256 computations. In the absence of high speed electronic computers which can evaluate the inverse of a 20×20 matrix in a matter of minutes, input-output theorists often have to resort to a procedure of 'iteration' to arrive at approximate answers to particular questions involving changes in final demand—but we will reserve discussion of the 'iterative' procedure for our analysis of Soviet planning.

Returning to our 2×2 matrix, suppose it was of the following order:

$$[1-A] = \begin{bmatrix} 1 & -0.5 \\ -0.25 & 1 \end{bmatrix}.$$

[18] This operation with matrices is quite analogous to the technique of division in elementary algebra. For example, in elementary algebra if we had the equation $(1-a)\,x=y$ and we wished to solve for x in terms of a and y, we would divide through by $(1-a)$ which is equivalent to multiplying y by the reciprocal of $(1-a)$, i.e.

$$x = \frac{y}{(1-a)} = (1-a)^{-1}y.$$

The derivation of the inverse matrix can be arrived at through use of *determinants.* (For a more detailed description of inverse matrix computation, see, Caroline Dinwiddy, *Elementary Mathematics for Economists,* Nairobi; Oxford University Press, 1967, chapter XVII.) The determinant, denoted Δ, of a 2×2 matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is $(a_{11}a_{22}-a_{21}a_{12})$, i.e. it is the difference between the cross products of the 4 elements of the matrix. Each element in the inverse, r_{ij}, is equal to the *cofactor* of the element a_{ji} (note that the subscripts are reversed) divided by the determinant of the matrix; where the cofactor of a_{ji} is obtained by evaluating the remaining determinant after row j and column i have been omitted from the original matrix. The sign $(-1)^{i+j}$ is then attached to the cofactor. Thus, all the cofactors of the Leontief (I—A) matrix must be positive since in all cases where the determinant after eliminating row j and column i is negative, i+j will be an odd sum with the result that $(-1)^{i+j}=-1$ and the cofactor becomes positive. In a 2×2 matrix we have the following cofactors:

$$A_{11}=a_{22} \qquad A_{12}=a_{12}$$
$$A_{21}=a_{21} \qquad A_{22}=a_{11}$$

Thus, if A_{ij} is defined as the cofactor of a_{ji}, then:

(2.1a) $$r_{ij}=A_{ij}/\Delta.$$

For example, the value of the determinant for the above 2×2 matrix is:

$\Delta=(1\times1)-(-0.5\times-0.25)=1-0.125=0.875.$

Equation (2.1a) above therefore yields:

$$r_{11} = \frac{1.0}{0.875} = 1.143$$

$$r_{21} = \frac{0.25}{0.875} = 0.286$$

$$r_{12} = \frac{0.50}{0.875} = 0.571$$

$$r_{22} = \frac{1.0}{0.875} = 1.143.$$

Consequently we obtain as the inverse matrix $(I-A)^{-1}$ of our 2×2 Leontief matrix the following:

$$(I-A)^{-1}=[R]=\begin{bmatrix} 1.143 & 0.571 \\ 0.286 & 1.143 \end{bmatrix}.$$

Note the elements r_{ij} of the inverse matrix are *not* merely the reciprocals of the elements of the (I—A) matrix, i.e. r_{12} is *not* equal to $\frac{1}{-a_{12}}$ or $-\left(\frac{1}{-0.5}\right)=+2.$

The computations are much more complex.

As a final check to see if our inverse matrix is correct, we should be able to multiply it by the original matrix to obtain an identity matrix since,

$$(I-A) \,.\, (I-A)^{-1} = I$$

or

$$[A \,.\, A]^{-1} = [I].$$

Thus,

$$(I-A) \,.\, (I-A)^{-1} = \begin{bmatrix} 1 & -0.5 \\ -0.25 & 1 \end{bmatrix} \,.\, \begin{bmatrix} 1.143 & 0.571 \\ 0.286 & 1.143 \end{bmatrix}$$

$$= \begin{bmatrix} (1.143-.143) & (.571-.571) \\ (-.286+.286) & (-.143+1.143) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [I].$$

Returning now to our algebraic input-output model, we see that in terms of the n linear equations, system (2.8) may be rewritten:

$$\text{(2.8a)} \qquad X_i = \sum_{j=i}^{n} r_{ij} \,.\, Y_j \qquad i=1, 2, \ldots, n$$

where,

r_{ij} is the element in the ith row and jth column of the inverted Leontief matrix $(I-A)^{-1}=R$. In economic terms, each element r_{ij} indicates the amount of commodity i that must be produced for each *unit* of final demand for the goods of sector j. The inverted Leontief matrix for our hypothetical 5×5 economy is the following:

$$(I-A)^{-1}=R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} \end{bmatrix} = \begin{bmatrix} 1.19 & .11 & .40 & .08 & .34 \\ 0 & 1.00 & 0 & 0 & 0 \\ .16 & .19 & 1.50 & .30 & .30 \\ .10 & .50 & .32 & 1.06 & .41 \\ .08 & .31 & .27 & .05 & 1.18 \end{bmatrix}$$

For example, $r_{13}=0.40$ indicates that in order to sustain each unit of final demand for manufactured goods 0.40 units of agricultural products must be produced.[19] Given this R matrix and using the data in Table 2.2, system (2.8a) may be written as follows:

$$\begin{aligned} X_1 &= 1.19(80) + .11(40) + .40(45) + .08(25) + .34(15) \\ X_2 &= 0(80) + 1.00(40) + 0(45) + 0(25) + 0(15) \\ X_3 &= .16(80) + .19(40) + 1.50(45) + .30(25) + .30(15) \\ X_4 &= .10(80) + .50(40) + .32(45) + 1.06(25) + .41(15) \\ X_5 &= .08(80) + .31(40) + .27(45) + .05(25) + 1.18(15). \end{aligned}$$

To check whether our R matrix is correct and our system internally consistent, we would calculate the X's from the above 5 equations and compare them with the original total outputs given in Table 2.2. The student can verify that these figures are identical.

[19] Since total output is always greater than or equal to total final demand, we note that all diagonal elements of the inverted Leontief matrix will be greater than or equal to 1.0. Under what conditions will the diagonal elements equal 1.0?

USING THE INVERTED MATRIX FOR PLANNING AND/OR PREDICTION

It can now readily be seen that by inserting any given, predicted, or planned final demand, $Y_{j\ j=1,\ldots,n}$ into the right-hand side of each equation of system (2.8a), one can determine the corresponding level of output, $X_{i\ i=1,2,\ldots,n}$, of commodity i that will be produced as a result of this level of demand.[20] Similarly, and more importantly, system (2.8a) can be used to measure the probable effects of any *change* in final demands on the total output of *all* sectors of the economy. To measure the effects of these changes in final demands we merely rewrite system (2.8a) as:

$$(2.8b) \qquad \Delta X_i = \sum_{j=1}^{n} r_{ij}\, \Delta Y_{j\ i=1,2,\ldots,n}$$

where the symbol Δ here stands for 'the change in' final demand, ΔY, or 'the change in' total output, ΔX. For example, if the final demands for the products of the manufacturing industry, sector 3 in Table 2.2, were *expected* to increase or were *planned* to increase by, say 10 units, in the next accounting or planning period, *ceteris paribus* we could determine how much additional total output would have to be produced by all 5 sectors by using the equations in system (2.8b). Thus, we are given $\Delta Y_3 = +10$ and we wish to determine ΔX_1, ΔX_2, ΔX_3, ΔX_4 and ΔX_5. The procedure would be as follows:

$$\begin{aligned}
\Delta X_1 &= 1.19\,(\Delta Y_1) + .11\,(\Delta Y_2) + .40\,(\Delta Y_3) + .08(\Delta Y_4) + .34\,(\Delta Y_5)\\
\Delta X_2 &= 0\,(\Delta Y_1) + 1.0\,(\Delta Y_2) + 0\,(\Delta Y_3) + 0(\Delta Y_4) + 0\,(\Delta Y_5)\\
\Delta X_3 &= .16\,(\Delta Y_1) + .19\,(\Delta Y_2) + 1.5\,(\Delta Y_3) + .30(\Delta Y_4) + .30\,(\Delta Y_5)\\
\Delta X_4 &= .10\,(\Delta Y_1) + .50\,(\Delta Y_2) + .32\,(\Delta Y_3) + 1.06(\Delta Y_4) + .41\,(\Delta Y_5)\\
\Delta X_5 &= .08\,(\Delta Y_1) + .31\,(\Delta Y_2) + .27\,(\Delta Y_3) + .05(\Delta Y_4) + 1.18\,(\Delta Y_5).
\end{aligned}$$

Since $\Delta Y_1 = \Delta Y_2 = \Delta Y_4 = \Delta Y_5 = 0$ in our simple example, we can eliminate the 1st, 2nd, 4th, and 5th terms on the right-hand side of the above five equations and arrive at our answer more directly. Thus,

$$\begin{aligned}
\Delta X_1 &= .40\Delta Y_3 = .40(10) = 4.0\\
\Delta X_2 &= 0\Delta Y_3 = 0(10) = 0\\
\Delta X_3 &= 1.5\ \Delta Y_3 = 1.5\ (10) = 15.0\\
\Delta X_4 &= .32\Delta Y_3 = .32(10) = 3.2\\
\Delta X_5 &= .27\Delta Y_3 = .27(10) = 2.7.
\end{aligned}$$

As a result of an increase in total *final demand* for manufactured products of 10 units, total *output* of agriculture (X_1) will rise by 4.0 units, output of the

[20] Note that each of the constants, r_{ij}, is a function of *all* the a_{ij}'s and if the production process of any one sector is altered due to technological advancement, better management, etc., more than one r_{ij} (in all probability quite a number of r_{ij}'s) will be affected. Consequently the assumption that the technical coefficients are constant in the shortrun is vital to the applicability of the input-output model.

extractive industry (X_2) will remain unchanged, and the total outputs of the manufactured (X_3), power (X_4), and transportation (X_5) sectors will rise by 15.0, 3.2, and 2.7 units respectively. Since each sector must purchase inputs from other sectors in order to produce more units of its own outputs, we see that the total *direct* and *indirect* effects of an initial increase in final demand will reverberate throughout the entire economy in a vast maze of economic interdependences until the combined increase in total output of all sectors of the economy is many times the magnitude of the initial stimulus. In short, if we are given, anticipate, or plan any change in final demands for any good or combination of goods, the input-output equations of system (2.8b) would portray the full direct and indirect impact of this change or these changes on all sectors of the economy.[21]

SOME EXTENSIONS OF THE BASIC MODEL

DETERMINATION OF EMPLOYMENT LEVELS

Before concluding this discussion of the mathematics of the basic input-output model, mention should be made of a few of the other important areas of economic analysis where the model can be extremely informative. For example, we might utilize techniques quite similar to those described above to estimate the impact of any change in final demand or total output on the level of total industrial employment in the economy. The industrial employment can be thought of as being distributed in certain proportions throughout all industries. In the household row of primary inputs in Table 2.2, we have a set of five figures representing the value of the labour input used by each of the five sectors. If these values are all written in terms of some average wage rate in the economy, then the relative magnitude of the various figures would correspond to the relative numbers of workers employed in that sector. For example, if the average annual wage rate in our hypothetical economy was equal to 0.001 units of value and if the household figures in Table 2.2 represented the total value of man-

[21] The precise meaning of 'direct' and 'indirect' impacts can be elucidated by means of a simple example. Suppose the final demand for automobiles dropped by 10 per cent. The direct impact of this change would naturally be a 10 per cent drop in the production of cars. However, since auto manufacturers use steel, rubber, cloth, etc., in producing their cars, they would demand less of these inputs and the *initial* indirect effect of this 10 per cent reduction in auto output would be a reduction in the steel, rubber, cloth and other interindustry outputs that are used as inputs in auto production. Finally, with their outputs also curtailed, the steel, rubber, and cloth industries will in turn demand less inputs from other industries. This iterative process of second, third, fourth, etc., round effects of the initial stimulus will continue until practically all sectors of the economy whose output is in any way connected with auto production have been affected. Thus, the total effect will be substantially greater than the initial direct impact of reduced auto production.

years employed by each sector,[22] then we could say agriculture is employing 40,000 man-years of labour (i.e. $40 \frac{1}{.001} = 40{,}000$), extractive industry, 5,000 man-years, and manufacturing, power, and transportation 6,000, 5,000, and 2,000 man-years respectively. Total employment in the five industrial sectors is thus at a level of 58,000 man-years. Since our input-output table is based on an accounting period of one year, we may refer to the total employment of 58,000 workers. In a manner analagous to the derivation of our technical coefficients of production, we can now derive a row vector of n *labour coefficients*, $l_{i\;i=1,\,2,\,\ldots,\,n,}$ each element of which depicts the number of workers (or man-years of employment) required to produce a *unit* of industry i's output. The labour coefficient is, therefore, calculated as follows for each industry:

$$l_i = \frac{L_i}{X_i}\quad {}_{i=1,\,2,\ldots,\,n}$$

where,

L_i is the level of employment in industry i, and

X_i is again the total output of industry i.

For example, the labour coefficients for our five sector economy would be the following:

$$l_1 = \frac{40}{125} = .32;\; l_2 = \frac{5}{40} = .125;\; l_3 = \frac{6}{100} = .06;\; l_4 = \frac{5}{75} = .066;\; l_5 = \frac{2}{50} = .04.$$

The level of employment in each industry is uniquely related to the amount of total output produced by that industry. Thus, to find the amount of labour employed in industry i, we merely multiply the corresponding labour coefficient l_i by the total output X_i of that sector. By summing the products of labour coefficients and total outputs of all industries throughout the economy, we can derive the following expression for *total* industrial employment:

$$L_T = \sum_{i=1}^{n} l_i X_i \tag{2.9}$$

where L_T represents total industrial employment in the economy. Similarly, a *change* in employment as a result of a *change* in total output can be expressed as:

$$\Delta L_T = \sum_{i=1}^{n} l_i \Delta X_i. \tag{2.9a}$$

Finally, since we know that $\Delta X_i = \sum_{j=1}^{n} r_{ij} \Delta Y_j$ from (2.8b), the change in employment as a consequence of any given, anticipated, or planned change in *final demand* can be calculated by substituting this relationship into (2.9a).

[22] A 'man-year' is defined as the average amount of hours worked by the average worker in one year's time. Thus, 100 man-years of labour could be interpreted as the equivalent amount of work performed by 100 men in one year's time or by one man in 100 year's time so long as the average work year remains unchanged.

Thus,

$$\Delta L_T = \sum_{i=1}^{n} l_i \left(\sum_{j=1}^{n} r_{ij} \, \Delta Y_j \right)$$

or

(2.9b) $$\Delta L_T = \sum_{i=1}^{n} \sum_{j=1}^{n} l_i \, r_{ij} \, \Delta Y_j \, .$$

For example, the change in total employment that would result directly and indirectly from our hypothetical increase in the final demand for the products of the manufacturing sector (i.e. $\Delta Y_3 = +10$) would be:

$\Delta L_T = l_1 r_{13} \Delta Y_3 + l_2 r_{23} \Delta Y_3 + l_3 r_{33} \Delta Y_3 + l_4 r_{43} \Delta Y_3 + l_5 r_{53} \Delta Y_3$
$= .32\,(.40 \times 10) + .125\,(0 \times 10) + .06\,(1.5 \times 10) + .062\,(.32 \times 10) + .04\,(.27 \times 10)$
$= 1.28 + 0 + .90 + .211 + .108$
$= 2.499.$

Thus, there will be increased employment opportunities for 2,499 workers as a result of a 10 unit or 22 per cent (10/45=.22) rise in final demand for manufactured products. It is interesting to note that the greatest *individual* sector impact on employment is *not* in the manufacturing sector (900 workers), as we might expect, but in the agricultural sector where 1,280 new jobs will be created. It is in unexpected circumstances like these that the real value of input-output analysis becomes strikingly evident.

BALANCE OF PAYMENTS ANALYSIS

The input-output model can be used also in the area of international trade to examine the approximate impact of any predicted or planned change in final demand or total output on the balance of payments (current account) position of the given economy. Since foreign trade is such an integral aspect of the structure of most African economies, any complete and comprehensive development plan must include some estimate of future balance of payments situations. We can use our row vector of intermediate imports in Table 2.2 and our column vector of total outputs to derive a row vector of *import* coefficients, $m_{i \; i=1,2,\ldots,n}$, one for each sector of the economy. Again, the procedure is exactly analagous to our derivation of labour coefficients, namely:

(2.10) $$m_i = \frac{M_i}{X_i} \quad {}_{i=1,2,\ldots,n}$$

where M_i is equal to the value of intermediate imports of sector i. Thus, the intermediate import coefficients for our five sector economy would be the following:

$$m_1 = \frac{15}{125} = .12;\; m_2 = \frac{0}{40} = 0;\; m_3 = \frac{10}{100} = .10;\; m_4 = \frac{30}{75} = .40;\; m_5 = \frac{5}{50} = .10.$$

Any change in total output or final demand will lead to an induced change in intermediate imports in accordance with the following two expressions:

(2.10a) $$\Delta M_T = \sum_{i=1}^{n} m_i \, \Delta X_i$$

and

(2.10b) $$\Delta M_T = \sum_{i=1}^{n} \sum_{j=1}^{n} m_i \, r_{ij} \, \Delta Y_j$$

where ΔM_T represents the change in the total value of intermediate imports. For example, as a result of our 10 unit increase in final demand for manufactured goods, induced intermediate imports will rise by:

$$\begin{aligned} \Delta M_T &= .12(4) + 0(0) + .10(15) + .40(3.2) + .10(2.7) \\ &= .48 + 0 + 1.5 + 1.28 + .27 \\ &= 3.53 \text{ units of value.} \end{aligned}$$

Unless exports are expanded, the increase in final demand could lead to balance of payments problems. It is to important questions like these that arise out of certain seemingly unrelated circumstances that planners must direct their attention if the economic plan is in any sense to be 'comprehensive'.

NON-HUMAN PRIMARY INPUTS

Finally, row vectors of capital and natural resource coefficients, representing the amounts of these factors used up, per unit of total output, denoted c_i and n respectively, can also be derived as above. Thus,

(2.11) $$c_i = \frac{C_i}{X_i} \quad {}_{i=1,\,2,\,\ldots,\,n}$$

and

(2.12) $$n_i = \frac{N_i}{X_i} \quad {}_{i=1,\,2,\,\ldots,\,n}$$

would be the relationships involved in the computation of capital and natural resource coefficients, and

(2.11a) $$\Delta C_T = \sum_{j=1}^{n} c_i \, \Delta X_i$$

or

(2.11b) $$\Delta C_T = \sum_{i=1}^{n} \sum_{j=1}^{n} c_i \, r_{ij} \, \Delta Y_j$$

and

(2.12a) $$\Delta N_T = \sum_{i=1}^{n} n_i \, \Delta X_i$$

or

(2.12b) $$\Delta N_T = \sum_{i=1}^{n} \sum_{j=1}^{n} n_i \, r_{ij} \, \Delta Y_j$$

would be the corresponding equations relating changes in the utilization of capital (ΔC_T) and natural resources (ΔN_T) required to achieve the increased outputs or final demands. For example, what will be the effect of our hypothetical 10 unit change in the final demand for manufactured goods on C_T and N_T? The crucial question then arises as to whether or not the economy has the necessary labour, capital and natural resources to achieve the increased output targets. If sufficient factors are unavailable, then the Ministry of Planning will have either to adjust its plan in accordance with these factors and material constraints, or induce managers and workers to put a little extra effort into their production activities. But we shall reserve further discussion of this problem until our study of the techniques of Soviet planning in the next chapter and in our discussion of 'activity analysis' and 'programming' for economic development in chapter 5.

Suggested Readings

1. THEORETICAL STRUCTURE OF INPUT-OUTPUT ANALYSIS

T. Barna (ed.), *The Structural Interdependence of the Economy*, Proceedings of an International Conference on Input-Output Analysis, Varenna, June 27–July 10, 1954 (New York, J. Wiley and Sons, Inc., 1956). Note articles by Leontief, Evans, and Morton.

Burgess Cameron, *Input-Output Analysis and Resource Allocation* (Cambridge, Cambridge University Press, 1968).

H. B. Chenery and P. G. Clark, *Interindustry Economics* (New York, J. Wiley and Sons, Inc., 1959), chapters 1–3.

Chiou-Shuang Yan, *Introduction to Input-Output Economics* (New York, Holt, Rinehart and Winston, Inc., 1969).

A. Ghosh, *Planning, Programming and Input-Output Models: Selected Papers on Indian Planning* (Cambridge, Cambridge University Press, 1968).

W. W. Leontief, *Input-Output Economics* (New York, Oxford University Press, 1966).

National Bureau of Economic Research, 'Input-Output Analysis: An Appraisal'. *Studies in Income and Wealth*, Vol. 18. (New York, Princetown University Press, 1955). Note especially contributions by Leontief, Evans, and Christ.

United Nations, Economic Commission for Latin America, 'The Input-Output Model', *Economic Bulletin for Latin America*, Vol. 1, No. 2. September 1956.

2. MATHEMATICAL AND COMPUTATIONAL TECHNIQUES

C. Dinwiddy, *Elementary Mathematics for Economists* (Nairobi, Oxford University Press, 1967), chapter XVII.

D. Gale, *The Theory of Linear Economic Models* (New York, McGraw-Hill, 1960).

G. Hadley, *Linear Algebra* (Massachusettes, Addison-Wesley, 1961), chapter 3.

K. Lancaster, *Mathematical Economics* (New York, Macmillan, 1968), chapter 6.

CHAPTER 3 Input-Output Analysis and Central Economic Planning

Having completed our task of explaining the analytical and mechanical nature of the basic input-output model, let us now see how the Ministry of Economic Planning in, say, a centralized socialist or 'command' economy might utilize the above techniques in constructing a comprehensive economic plan. We shall assume that the data contained in Table 2.2 of the previous chapter represents the structural make-up of this economy in some period t and that this data forms the basis upon which the plan for period t+x is to be drawn up. Although the following illustration will necessarily be of a highly simplified nature, it is hoped that the student will benefit in at least two ways from this hypothetical planning problem, in that (1) he will form a sound idea of the nature of the planning process in an advanced collectivist economy, and (2) he will see how input-output analysis can serve as a valuable tool in the construction of *consistent* and *comprehensive* economic plans. Again the student is reminded that although the model is more amenable to some degree of centralized co-ordination and control, the existence of a collectivist economy is by no means a prerequisite for reaping the full benefits of the input-output approach to development planning.[1]

A HYPOTHETICAL PLANNING PROBLEM

FINAL DEMAND TARGETS

Suppose that on the basis of the data contained in Table 2.2 for period t, the Ministry of Planning decides that in period t+x it would like to see the economy

[1] In the Appendix to this chapter we will discover the extent to which input-output tables have been used in the various economies of the world, both developed and underdeveloped. Then, in chapter 4, we shall see specifically how input-output can be applied to problems in developing countries.

achieve the following set of *target* goals for final demand (i.e. GDP):[2]

1. a *ten per cent* or 8 unit increase in the demand for agricultural goods of which household food consumption is to rise by 3 units and agricultural exports by 5 units,
2. a *five per cent* or 2 unit risc in the demand for the mineral products of the extractive industry all of which is to be exported to foreign countries,
3. a *twenty per cent* or 9 unit rise in the demand for manufactured goods of which 3 units will be allocated for household consumption and 6 units for government investment in state operated enterprises,
4. an *eight per cent* or 2 unit rise in the final demand for power all of which is to be consumed by households, and
5. a *twenty per cent* or 3 unit rise in the final demand for transportation all of which is to be in the form of investment demand, e.g. by building an extension to the present railroad network.

SOME IMPORTANT PLANNING QUESTIONS

On the basis of these target goals, the Ministry might utilize its input-output model to answer the following four important planning questions:

1. What level of *total output*, X_i, will be required in each of the five producing sectors of the economy to achieve the given demand targets?
2. What will be the magnitude and direction of *interindustry product flows*, x_{ij}, to satisfy the resultant material input demands of the five sectors?
3. How will *intermediate imports*, M_i, be affected, and will the target demands present any balance of payments problems?
4. What effect will these demand targets have on the level of *primary inputs* (labour, capital, and natural resources) that will have to be utilized in the economy in order to carry out successfully the aims of the overall plan?

Once these questions have been answered a new input-output table representing a consistent and comprehensive plan for the period $t+x$ can be drawn up. But before turning to this task let it be clear that in actual practice the planning problem is obviously much more involved and many additional relevant questions need to be asked. Among others, the following would certainly stand out:

(a) How is the plan to be financed?
(b) What criteria should be involved in drawing up investment projects?
(c) Should agriculture or industry be given top priority?

[2] More often than not, as in the case of the Soviet Union, collectivist economic plans have been draw up on the basis of target goals for *total output* rather than final demand. In non-collectivist developing economies like those in Africa, plans are usually drawn up on the basis of consumption, investment, export, etc., targets (i.e. on the basis of final demand objectives). Consequently, we will utilize the final demand approach in this section and reserve discussion of the total output approach to the section on Soviet economic planning.

(d) Do the economy and existing industries have the necessary resources and capacity to produce the required outputs? If not, what should be done about it, i.e. should the plan be made less ambitious, and if so, how?

Questions (a) to (c) are best treated in text books on economic development [3] and would lead us far astray if we attempted to deal with them here. Our present purposes will be served most directly, however, if we concentrate on the above four *basic* questions and deal with some of the relevant issues of capacity and resource constraints posed by question (d) briefly in the next section and more completely in chapter 5.

Question 1: How Will Total Output be Affected?

In order to ascertain the effects of any changes in final demand on total outputs in all sectors of the economy, we utilize system (2.8a) of our input-output model:

$$X_i = \sum_{i=1}^{N} r_{ij} Y_j \quad {\scriptstyle i=1,\,2,\,\ldots,\,n.}$$

The Ministry of Planning has decided that final demands for the goods of sectors 1 through 5 are to rise by 8 units, 2 units, 9 units, 2 units, and 3 units respectively, i.e. $\Delta Y_1=8$, $\Delta Y_2=2$, $\Delta Y_3=9$, $\Delta Y_4=2$, and $\Delta Y_5=3$. We know from Table 2.2 that in period t total final demands were 80, 40, 45, 25, and 15 for these same five sectors. Consequently, in period t+x the planned target final demands will be as follows:

$$\begin{aligned}
Y_{1(t+x)} &= Y_{1(t)} + \Delta Y_1 = 80 + 8 = 88 \text{ units}\\
Y_{2(t+x)} &= Y_{2(t)} + \Delta Y_2 = 40 + 2 = 42 \text{ units}\\
Y_{3(t+x)} &= Y_{3(t)} + \Delta Y_3 = 45 + 9 = 54 \text{ units}\\
Y_{4(t+x)} &= Y_{4(t)} + \Delta Y_4 = 25 + 2 = 27 \text{ units}\\
Y_{5(t+x)} &= Y_{5(t)} + \Delta Y_5 = 15 + 3 = 18 \text{ units}
\end{aligned}$$

where the subscripts in parentheses, t and t+x, indicate the two successive periods. Given the r_{ij}'s of our inverse matrix [4] and the above vector of final demands for period t+x we calculate total output as follows:

$$\begin{aligned}
X_{1(t+x)} &= \sum_{j=1}^{5} r_{1j} Y_{j(t+x)}\\
&= 1.19(88)+.11(42)+.40(54)+.08(27)+.34(18)\\
&= 104.72+4.62+21.60+2.16+6.12\\
&= 139.22 \text{ units of value;}\\
X_{2(t+x)} &= \sum_{j=1}^{5} r_{2j} Y_{j(t+x)}\\
&= 0(88)+1.00(42)+0(54)+0(27)+0(18)\\
&= 42 \text{ units of value;}
\end{aligned}$$

[3] For example, see Gerald Meier, *Leading Issues in Development Economics* (New York, Oxford University Press, 1970).

[4] See page 31 of chapter 2 for the numerical elements of our hypothetical inverse matrix.

$$\begin{aligned} X_{3(t+x)} &= \sum_{j=1}^{5} r_{3j} Y_{j(t+x)} \\ &= .16(88)+.19(42)+1.50(54)+.30(27)+.30(18) \\ &= 14.08+7.98+81.00+8.10+5.40 \\ &= 116.56 \text{ units of value;} \end{aligned}$$

$$\begin{aligned} X_{4(t+x)} &= \sum_{j=1}^{5} r_{4j} Y_{j(t+x)} \\ &= .10(88)+.50(42)+.32(54)+1.06(27)+.41(18) \\ &= 8.80+21.00+17.28+28.62+7.38 \\ &= 83.08 \text{ units of value;} \end{aligned}$$

$$\begin{aligned} X_{5(t+x)} &= \sum_{j=1}^{5} r_{5j} Y_{j(t+x)} \\ &= .08(88)+.31(42)+.27(54)+.05(27)+1.18(18) \\ &= 7.04+13.02+14.58+1.35+21.24 \\ &= 57.23 \text{ units of value.} \end{aligned}$$

In order to fulfil the target demands, total output of agricultural products will have to rise by 11.3 per cent from 125 units to 139.22 units in period t+x. Since none of the five sectors use the products of the extractive industry as intermediate inputs, total output will always equal total final demand for sector 2, i.e. $X_{2(t+x)}=Y_{2(t+x)}=42$.

Manufacturing output must rise by 16.56 per cent from 100 to 116.56 units; the output of the power industry must increase by 10.7 per cent from 75 to 83.08 units; and the transport sector must expand its output 14.5 per cent from 50 to 57.23 units in the planned period t+x. This completes the first step in constructing the plan and provides us with the necessary information for answering question 2.

Question 2: What is the Required Magnitude and Direction of Intermediate Input Flows?

In a centrally controlled economy, the state not only sets final demand and total output targets for all sectors, but must also inform the production managers of each industry how they are to produce their required outputs. The comprehensive collectivist plan, therefore, must also include instructions to all production managers informing them how much input they are to obtain from the other sectors of the economy and how much of their total output is to be delivered for intermediate use in these other sectors. In short, the Ministry of Planning must also compute all the necessary x_{ij}'s of the transaction matrix for period t+x. Having calculated total output, the determination of intermediate input flows becomes a relatively simple task with the aid of the 'A' matrix of technical coefficients. Thus, according to system (2.1), each

$$x_{ij\,(t+x)} = a_{ij}\, X_{j\,(t+x)} \qquad \begin{matrix} i=1, 2, \ldots, 5 \\ j=1, 2, \ldots, 5, \end{matrix}$$

Using the data for our *fixed* technical coefficients in Table 2.4 of chapter 2, we can arrive at the following results:[5]

Column 1

$$x_{11} = a_{11}X_1 = .12(139.22) = 16.7$$
$$x_{21} = a_{21}X_1 = 0(139.22) = 0$$
$$x_{31} = a_{31}X_1 = .08(139.22) = 11.1$$
$$x_{41} = a_{41}X_1 = .04(139.22) = 5.6$$
$$x_{51} = a_{51}X_1 = .04(139.22) = 5.6$$

Column 2

$$x_{12} = a_{12}X_2 = 0(42) = 0$$
$$x_{22} = a_{22}X_2 = 0(42) = 0$$
$$x_{32} = a_{32}X_2 = 0(42) = 0$$
$$x_{42} = a_{42}X_2 = .38(42) = 15.96$$
$$x_{52} = a_{52}X_2 = .25(42) = 10.50$$

Column 3

$$x_{13} = a_{13}X_3 = .20(116.56) = 23.3$$
$$x_{23} = a_{23}X_3 = 0(116.56) = 0$$
$$x_{33} = a_{33}X_3 = .25(116.56) = 29.14$$
$$x_{43} = a_{43}X_3 = .15(116.53) = 17.48$$
$$x_{53} = a_{53}X_3 = .15(116.53) = 17.48$$

Column 4

$$x_{14} = a_{14}X_4 = 0(83.08) = 0$$
$$x_{24} = a_{24}X_4 = 0(83.08) = 0$$
$$x_{34} = a_{34}X_4 = .20(83.08) = 16.6$$
$$x_{44} = a_{44}X_4 = 0(83.08) = 0$$
$$x_{54} = a_{54}X_4 = 0(83.08) = 0$$

Column 5

$$x_{15} = a_{15}X_5 = .20(57.23) = 11.45$$
$$x_{25} = a_{25}X_5 = 0(57.23) = 0$$
$$x_{35} = a_{35}X_5 = .10(57.23) = 5.72$$
$$x_{45} = a_{45}X_5 = .30(57.23) = 17.16$$
$$x_{55} = a_{55}X_5 = .10(57.23) = 5.72.$$

[5] Note, all the x_{ij}'s are for planned period t+x. We have therefore omitted the parenthesized subscripts in the following figures to avoid symbolic confusion. Note also that each group of five calculations corresponds to the elements in the column vectors of the new transactions matrix for t+x, i.e. group 1 = column 1, the intermediate inputs of sector 1.

Consequently, the new transactions matrix for target period $t+x$ will be as follows:

	Agriculture	*Extractive industry*	*Manufacturing*	*Power*	*Transportation*
Agriculture	16.7	0.00	23.30	0.0	11.45
Extractive industry	0.0	0.00	00.00	0.0	0.00
Manufacturing	11.1	0.00	29.14	16.6	5.72
Power	5.6	15.96	17.48	0.0	17.16
Transportation	5.6	10.50	17.48	0.0	5.72

The officer in charge of, say, manufactured products would then receive instructions from the central planning agency that he is to purchase 23.3 units of agricultural products,[6] to set aside 29.14 units of his own products,[7] to obtain 17.48 units of power to run his factories and to make use of the transportation network to the extent of 17.48 units of value. Similarly, the manufacturing sector is instructed to deliver 11.1 units of its product to the agricultural sector, 29.14 units to itself, 16.6 units to the power industry, and 5.72 units to the transport sector. The production managers of the other four sectors would also receive their corresponding instructions and a vast network of interrelated and consistent interindustry flows of inputs and outputs will be established.

Question 3: Will there be any Serious Balance of Payments Problems?
Not only will sectors have to engage in substantial interindustry transactions to achieve their required output goals but they will also have to purchase the necessary foreign imports of intermediate goods as dictated by their particular production processes. Thus, central planners must compute the intermediate import requirements for two specific reasons: (1) to inform production managers how many (and presumably, from where) imports are to be obtained, and (2) to ascertain whether or not the plan might lead to any significant balance of payments problems. Using system (2.10) we can calculate the required intermediate imports as follows:

$$M_{i(t+x)} = m_i X_{i(t+x)} \quad i=1, 2, \ldots, 5.$$

Thus,

$$M_{1(t+x)} = .12(139.22) = 16.70$$
$$M_{2(t+x)} = 0(42) = 0$$
$$M_{3(t+x)} = .10(116.56) = 11.66$$
$$M_{4(t+x)} = .40(83.08) = 33.23$$
$$M_{5(t+x)} = .10(57.23) = 5.72.$$

[6] Recall that we are speaking of 'units of value' at constant prices.

[7] For accounting purposes, the manufacturing sector is assumed to have 'purchased' 29.14 units of its own product even though no actual payment has been made.

Intermediate imports, therefore, will rise by 7.31 units from a total of 60 in period t to 67.31 in period t+x. Since final demand exports are planned to rise by 7 units (i.e. 5 units of agricultural and 2 units of mineral exports), no immediate balance of payments problem is involved in the overall plan so long as the state is able to effectively curb additional final demand imports.[8]

Question 4: How Much Labour, Capital, and Natural Resources does the Plan Require?

Questions 1 through 3 have provided us with enough information to fill in the entries for the total output vector, the transactions quadrant and the intermediate import vector of our new input-output table. Our centrally determined targets for final demands give us the data to be contained in the final use quadrant. In order to fill in the entries in the third principal quadrant of the table, the value added quadrant, however, we must ascertain the level of primary inputs that will be needed to adequately satisfy all elements of the plan. Input-output equations (2.9), (2.11) and (2.12) must therefore be utilized. For example, the impact of the plan on the level of industrial employment would be calculated according to equation (2.9). Thus, we use:

$$L_{i(t+x)} = l_i X_{i(t+x)} \quad {}_{i=1,\,2,\ldots,\,5}$$

to arrive at the following results:

$$\begin{aligned}
L_{1(t+x)} &= l_1X_{1(t+x)} = .32(139.22) &&= 44.55\\
L_{2(t+x)} &= l_2X_{2(t+x)} = .125(42) &&= 5.25\\
L_{3(t+x)} &= l_3X_{3(t+x)} = .06(116.56) &&= 6.99\\
L_{4(t+x)} &= l_4X_{4(t+x)} = .066(83.08) &&= 5.48\\
L_{5(t+x)} &= l_5X_{5(t+x)} = .04(57.23) &&= 2.29
\end{aligned}$$

which yields a figure for total industrial employment $L_{T(t+x)}$ of 64,960 workers, an increase of 6,960 jobs over period t.

The capital use requirements for the plan are given by equation (2.11):

$$C_i = c_i X_i \quad {}_{i=1,\,2,\ldots,\,5}$$

Therefore, we obtain:

$$\begin{aligned}
C_{1(t+x)} &= c_1X_{1(t+x)} = .04(139.22) &&= 5.57\\
C_{2(t+x)} &= c_2X_{2(t+x)} = .075(42) &&= 3.15\\
C_{3(t+x)} &= c_3X_{3(t+x)} = .05(116.56) &&= 5.83\\
C_{4(t+x)} &= c_4X_{4(t+x)} = .16(83.08) &&= 13.29\\
C_{5(t+x)} &= c_5X_{5(t+x)} = .08(57.23) &&= 4.58
\end{aligned}$$

which yields a figure of 32.42 units of value, $C_{T(t+x)}$, representing total depreciation and/or payments for the use of capital equipment in accordance with the overall objectives of the plan—an increase of 3.42 over period t.

[8] Note, however, that there will still be a small 'deficit' in overall trade balance on current account.

Finally, the natural resource utilization requirements are given by equation (2.12.) as

$$N_i = n_i X_i \quad {}_{i=1,\,2,\ldots,\,5}$$

which yields the following figures for period (t+x):

$$N_{1(t+x)} = n_1 X_{1(t+x)} = .08(139.22) = 11.14$$
$$N_{2(t+x)} = n_2 X_{2(t+x)} = .05(42.00) = 2.10$$
$$N_{3(t+x)} = n_3 X_{3(t+x)} = .01(116.56) = 1.17$$
$$N_{4(t+x)} = n_4 X_{4(t+x)} = .08(83.08) = 6.65$$
$$N_{5(t+x)} = n_5 X_{5(t+x)} = .04(57.53) = 2.30.$$

Thus, the amount of natural resources, $N_{T(t+x)}$, used up during the planning period will amount to 23.36 units of value at constant prices—a net increase of 2.36 units over the previous period.

The crucial question now facing the Ministry of Planning is whether these additional labour, capital and natural resource requirements are readily available in the economy, or, is the economy in some way 'constrained' by either a lack of sufficient primary or material inputs? If the latter is the case, then planners are faced with two alternative courses of action: (1) they can attempt to operate within the context of the given constraints by trying to 'readjust' or 'balance' their target objectives so as not to overestimate their production capabilities, or (2) they may disregard the constraints and still try to achieve their original target goals by a process of 'belt tightening' in which the same outputs are attained with fewer inputs, i.e. by trying to reduce the technical and primary input coefficients through improved production methods, longer work hours, etc. We shall try to analyse both of these approaches in the remainder of this chapter with particular reference to the operational methodology of Soviet planning.

THE COMPLETED COMPREHENSIVE PLAN: A NEW INPUT-OUTPUT TABLE

Before turning to these important questions, however, let us summarize the results of our hypothetical central planning problem by piecing together the various parts of the preceding analysis into a consistent input-output table which would represent the comprehensive plan for our hypothetical economy in period t+x. The student should now be able to interpret the precise meaning of all entries in the following table.[9]

[9] For example, what is the new level of gross domestic product? Does the value added income approach correlate with the final demand product approach to the computation of GDP? Note that figures might not add up exactly since calculations were only carried out to two decimal places. Also entries in the direct factor purchases quadrant (e.g. taxes and government employment) are assumed to expand roughly in proportion to the percentage growth in final demand (i.e. by approximately 11 per cent). Obviously, this is a highly simplified and unrealistic assumption and in actual practice projections of tax receipts, in particular, would be much more involved.

Table 3.1

The Complete Input-Output Table of our Centrally Planned Economy for Period t+ ×

Using Sectors Inputs / *Producing Sectors Outputs*	*Intermediate Use*						*Final Use (Demand)*					
							Private	*Govern-ment*	*Govern-ment*	*Foreign*		
	Agricul-ture	Extrac-tive industry	Manu-facturing	Power	Trans-porta-tion	*Total Inter-mediate Demand*	House-hold con-sumption	Invest-ment	Non-Invest-ment expendi-ture	Exports	*Total Final Demand*	*Total Output*
Agriculture	16.70	0	23.30	0	11.45	51.45	38	10	5	35	88	139.22
Extractive industry	0	0	0	0	0	0	0	10	0	32	42	42.00
Manufacturing	11.10	0	29.14	16.60	5.72	62.56	18	26	5	5	54	116.56
Power	5.60	15.96	17.48	0	17.16	56.20	7	10	10	0	27	83.08
Transportation	5.60	10.50	17.48	0	5.72	39.30	5	11	2	0	18	57.23
Total Purchases	39.00	26.46	87.40	16.60	40.05	209.51	68	67	22	72	229	
Imports	16.70	0	11.66	33.23	5.72	67.31	5	5	0	0	10	67.31 (77.31)
Government (Taxes)	22.28	5.05	3.50	7.83	2.29	40.95	(38.85)	(0)	(0)	(22.2)	(61.05)	40.95 (102)
Household (Labour)	44.53	5.25	6.99	5.48	2.29	64.96	1.11	0	13.32	0	14.43	79.39
Capital	5.57	3.15	5.83	13.29	4.58	32.48	0	0	0	0	0	32.48
Natural resources	11.14	2.10	1.17	6.65	2.30	23.36	0	0	0	0	0	23.36
Value Added	83.52	15.55	17.49	33.25	11.46	161.69	1.11	0	13.32	0	14.43	176.12
Total Inputs	139.22	42.0	116.56	83.08	57.23	438.50	74.11	72	35.32	72	243.43	681.93

CENTRAL PLANNING IN THE SOVIET ECONOMY: SOME GENERAL CONSIDERATIONS

While the planning of industrial supply in the Soviet economy is operationally and methodologically quite similar to the input-output approach, it must be stressed at the outset of this section that the USSR, and most of the other Soviet-type economies of Eastern Europe, do *not* possess input-output tables *per se*. This is the case even though the principal operational objective of Soviet planning is the achievement of a *material balance* between the amount supplied and demanded for every commodity on the basis of interindustry flows of inputs and outputs plus final demands. However, as we shall presently discover, the procedural process of matching supply and demand is carried out in a manner which is much less sophisticated than that of input-output mathematics even though Soviet planners do make extensive use of 'input norms', (which are quite analogous to the technical coefficients of interindustry economics) and to the iterative procedure of calculating the amount of all material inputs necessary to achieve the planned output targets.

This iterative method is intended to arrive at approximately the same results as that of the more sophisticated input-output process of matrix inversion without the necessity of using high-speed electronic computers. As we shall shortly see, this rather crude approach to matching inputs and outputs can lead to serious calculating errors if not adequately carried out. But, before analysing material balances and the iterative solution, let us briefly review the nature of the central planning process in the Soviet Union in order to provide a basic background for our following discussion.

THE NATURE OF THE SOVIET PLANNING PROCESS[10]

The Soviet plan is primarily a short term exercise with the main emphasis being

[10] The primary source of information for this section is Herbert S. Levine, 'The Centralized Planning of Supply in the Soviet Union', in Franklyn D. Holzman (ed.), *Readings on the Soviet Economy* (Chicago, Rand McNally and Co., 1962), pp. 329-354.

Recent events in the Soviet Union indicate a marked change in the attitude of the new leadership regarding the adequacy of (a) the complete centralized planning of supply and (b) the total output approach to central planning. There appears now to be a marked tendency towards a gradual adoption of the market oriented approach in the Soviet economy with production decisions being carried out on a more decentralized basis than ever before. For example, beginning on 1 April 1965 more than 400 consumer goods factories adopted a new system of production based on market demands. It is understood that this new system has since been extended gradually to a larger segment of the economy, including heavy industry. In effect, this fundamental reform of the Soviet economy will make certain output decisions much more responsive to the supply and demand mechanism of market economies rather than solely to the subjective priorities of central planners. Thus the following description of Soviet planning techniques in general and the method of material balances in

placed on the construction of annual (one year) plans.[11] The annual plan contains detailed targets for industrial and agricultural outputs, capital construction, primary inputs, and intermediate material flows. The general procedural process in drawing up the plan is roughly one in which a set of general instructions flow down the planning hierarchy (i.e. from the Council of Ministers to the various production units) followed by a reverse upward flow of information and suggestions from the botton. This is followed by co-ordination of material balances at the top and the issuance of a fairly detailed plan. In the last stage of the process, the plan again flows down the hierarchy and is put into the thorough detail necessary for operational purposes. The following four stages summarize the basic chronology of the annual plan.[12]

1. The *first stage* of the planning process consists of a statistical analysis of the previous year plus some estimates of the probable data for the planned year. The purpose of this statistical work is essentially to aid in the construction of the 'control figures' (stage two) by uncovering temporary 'bottlenecks' which should be given greater attention in the planned year and by locating possible additional sources of increased output.
2. The *second stage* is the drawing up of control figures.[13] These constitute the preliminary outlines of the forthcoming economic plan. They consist of a set of *aggregate output targets* for a dozen or so of the most important groups. They also contain some major investment targets. Their purpose is to serve as guideposts to the lower economic units in the construction of the annual plan.[14] We thus immediately note that in the Soviet Union the plan begins not with a set of final demands as in our input-output example but with a number of given *production targets* for *total output.*
3. The *third* and *key* stage of the Soviet plan is the one in which the plan comes back up the hierarchy and is co-ordinated on a national scale at the top. It is then confirmed by the government and becomes the operational law of the economy for the coming year. It is during this third stage that

particular is no longer strictly correct. However, since it is not our intention in this chapter to engage in an extensive analysis of the contemporary Soviet economy but rather to provide an illustration of one of the most well known real world examples of central planning, we will restrict our discussion to the centrally planned Soviet economy of the 1950s.

[11] Although there is no detailed operational 5 or 7 year plan in the Soviet Union, the annual plans are constructed within the context of a general set of medium range objectives.

[12] Levine, op. cit., pp. 156-162.

[13] Following the administrative reorganization of 1957, annual control figures were replaced by longer range output targets.

[14] It is at this point that we recognize the fact that in a centralized economy, 'planner's preferences' (in the USSR the 'planners' are actually the political leaders and it is they who make the final decision) take priority over 'consumer preferences'. The political leaders rather than the consumers determine the major directions that the economy will take in the coming year.

production managers calculate their individual material and primary input needs on the basis of their instructed target outputs. Since each firm must know how much of each input it will need per unit of output, it must draw up a set of 'input norms' similar to the column vector of a_{ij}'s of our input-output technology matrix. The guiding principle that underlies the working out of input norms is that they must aid in the constant struggle to increase economic output by forcing the spread of technical progress and the economizing of material resources. To accomplish this, these norms or coefficients must embody the achievements of the leading (i.e. most efficient) firms, but they must also be attainable by the average firm. Thus, the actual norm is usually established somewhere between the average and the best. Although these norms are originally worked out by each firm, they are always inspected by technical bureaus up the bureaucratic line and are frequently changed by higher organs during the construction of the annual economic plan in order to match available inputs with planned outputs.

After calculating its input requirements on the basis of its output targets (again, analagous to our $\sum_{i=1}^{N} a_{ij}X_j$), each enterprise sends its input orders back up the hierarchy. It is at this point in the plan that the complex process of material balancing takes place since input demands usually exceed output supplies (due to 'padding' of orders by production managers who attempt to assure the achievement of their targets). But we will discuss this crucial aspect of the plan in the next section.

4. *Stage four*, the final stage in the construction of the plan, consists once again in the bringing down of the plan to the enterprise and its transformation into detailed operational form. The complete plan is then drawn up and input-output instructions are sent to all production ministries in the economy. The plan is then put into action.

THE METHODOLOGY OF MATERIAL BALANCES

The 'material balance' is at the core of Soviet planning. It represents an attempt during the third stage of the planning process to match the total demand (both intermediate and final) for every major material and primary resource against its available supply. The crucial problem in the material balance technology is how are the planned sources and distributions of all inputs and outputs brought into balance when at first there is an imbalance? In terms of our input-output methodology, this problem consists in discovering how any change in the output target of one product will affect the required outputs of all other products. In the Soviet Union, the usual direction of any imbalance is that the demand for a product is greater than the originally planned supplies. And this is to be expected, since production managers fearing reprisals if they do not achieve

their output targets will often 'pad' their orders for input materials, i.e. they will request an amount greater than that dictated by the average progressive input norm of their particular industry. It is then the role of the USSR *Gosplan* and of the Planning Commissions of other Soviet-bloc countries to try to correct any imbalance and to draw up a final detailed plan that is internally consistent. Let us see what steps are taken to correct these imbalances.

Material balances can be thought of as an interlocking set of relationships which can be arranged as an input-output table with as many rows and columns as there are balances. Any structural imbalances can then be attacked in one of two ways. Let us suppose that the normal case of excess demand prevails.

Increasing Supplies of Deficit Commodities

First, planners can attempt to correct this divergence by *increasing* the *supply* of all deficit commodities. There are three possible sources of increasing the supply: (1) by drawing down whatever surplus stocks from previous years are still available, (2) by importing the deficit commodity, and (3) by increasing current production. It is the last of these three approaches which forms the major supply effort in the Soviet economy. Increased current production can be achieved within the planned capacity by better or more intensive use of existing plant and equipment. Alternatively, new capacity might be created by speeding up planned investments.

To the extent that an imbalance is corrected by increasing the supplies of deficit commodities, planners must take into account the fact that if the output of any commodity is to be expanded beyond planned targets then the output of all inputs used in the production of this good must also be expanded accordingly. In input-output terminology, this balancing process might be described as follows:

$[A]=$ matrix of technical coefficients or, as in the USSR, 'input norms'.
$\bar{X}_s=$ column vector of originally planned supplies, i.e. production (total output) targets.
$\bar{X}_d=$ column vector of total calculated demands both intermediate and final.
$\bar{Y}_d=$ column vector of planned final demands.

Suppose that there are a number of discrepancies between originally planned supplies and requested demands such that total intermediate and final demands exceed planned supplies.

Then, we would have the following situation:

$$[A]\,\bar{X}_s + \bar{Y} \leq \bar{X}_d,$$

where

$$\bar{X}_d \geq \bar{X}_s.$$

Now let

$\Delta X_s =$ the unknown vector of changes in $\bar{X}_s$

necessary to arrive at a consistent set of material balances. Since this increased supply will have to be large enough to satisfy a further increase in intermediate demand as well as to fill in the existing output gap, we must set,

$$\Delta X_s = \bar{X}_d - \bar{X}_s + [A]\Delta X_s,$$

where the last term represents the resultant increase in intermediate demands. Rewriting the expression, we obtain:

$$\Delta X_s - [A]\Delta X_s = \bar{X}_d - \bar{X}_s \text{ or, } [I-A]\Delta X_s = \bar{X}_d - \bar{X}_s$$

which can be rewritten as,

$$\Delta X_s = (I-A)^{-1}(\bar{X}_d - \bar{X}_s).$$

Thus, the computation of the necessary supply increase, ΔX_s, to bring about a material balance would be quite straightforward with the aid of the inverse matrix of our input-output analysis. Unfortunately, the Soviet Union, as we have already pointed out, does not possess such a formal table and must resort to a much cruder and less accurate mathematical procedure to arrive at a set of material balances.[15] This procedure is called the 'iterative method' of calculating material balances.

The iterative solution consists in the computation of the total effects (both direct and indirect) of any change in planned outputs $(\bar{X}_d - \bar{X}_s)$ necessary to correct a material imbalance. It is a process whereby the increase in the planned output of one commodity is followed in the first round, or 'iteration', by an increase in the output of all inputs into that commodity, followed in the second round by the increase in the output of all the inputs into the inputs, and so on down the line with all calculations being based on the set of 'input norms':

$$a_{ij} \quad i=j=1, 2, \ldots, n$$

Mathematically, it would take an infinite number of such steps, or iterations, before one could arrive at a perfectly consistent set of material balances that would result if $(I-A)^{-1}$ were employed.[16]

For example, suppose we let $\bar{X}^{(o)}$ represent a vector of initial divergences between $\bar{X}_d$ and $\bar{X}_s$, i.e. $\bar{X}^{(o)} = (\bar{X}_d - \bar{X}_s)$. On the basis of this vector, the input requirements for all industries could be calculated and then summed up for the whole economy. If this vector of summations is then added to the original $\bar{X}^{(o)}$, we obtain the 'first round' effects of the change in planned outputs. Thus,

$$\bar{X}^{(1)} = [A]\bar{X}^{(o)} + \bar{X}^{(o)} \tag{3.1}$$

[15] Since 1956, Russian economists have begun to give more attention to the possibilities of adapting input-output techniques to their balancing methodology. However, bureaucratic inertia and especially an underlying ideological opposition to input-output mathematics as being 'anti-Marxist' has greatly slowed down the introduction of this powerful planning tool into the Soviet economy.

[16] This is so because, letting (A) = matrix of input norms, $(I-A)^{-1} = (I + A + A^2 + \ldots A^n)$ where $n \to \infty$.

where $\bar{X}^{(1)}$ is the column vector of total first round effects of $\bar{X}^{(o)}$ on all other outputs when initial intermediate demands, $[A]\bar{X}^{(o)}$, have been considered. The 'second round' effects would then be calculated as:

$$\bar{X}^{(2)}=[A]\bar{X}^{(1)}+\bar{X}^{(o)}. \tag{3.2}$$

Similarly, for the 'third round', we obtain

$$\bar{X}^{(3)}=[A]\bar{X}^{(2)}+\bar{X}^{(o)}. \tag{3.3}$$

It can be shown that every new set of additional gross output targets, $\bar{X}^{(n)}$, obtained in the above manner would come closer to the perfectly consistent set of targets (ΔX_s in the previous example) that could be calculated by direct matrix inversion, i.e.

$$\underset{n\to\infty}{\bar{X}^{(n)}}=(I-A)^{-1}(\bar{X}_d-\bar{X}_s)=\Delta X_s.$$

The problem arises, however, when we realize that if this iterative process is not carried on long enough, the material balances may be grossly inaccurate.[17] Moreover, this problem is acute in the Soviet balancing methodology where both the great number of planned outputs and the absence of high-speed electronic computers physically precludes the computation of more than two or three iterations. For example, just to calculate the first round effects of changes in the planned outputs of, say, 100 commodities would required 10,000 computations.[18] When it is realized that there are more than 500 centrally allocated commodities in the Soviet Union, it becomes evident that no more than two or three iterations is within the physical capacity of Soviet planners.

Reducing the Demand for the Deficit Commodity

The second approach to solving the material balance problem consists in the attempt to bring about a *reduction* in the *demand* for deficit commodities. It is the usual Soviet policy that any reduction in demands should initially be achieved within the framework of the originally planned outputs, i.e. producers are asked to use less of the deficit commodities than they originally requested while still producing the *same* output. Intermediate demand reduction is to be accomplished by promoting increased efficiency in the use of the deficit material either by economizing ('belt tightening') or by adopting the production techniques of the most efficient firms. In short, production managers are requested to find some means of *reducing* their *input norms.*

[17] Actually, since the indirect effects become quite small after the first few iterations, a fairly good approximation to the correct solution can normally be arrived at with six to ten iterations.

[18] This is so because if there are n centrally allocated commodities, then to calculate the *direct* input requirements of a single output change, you need n multiplications. But to calculate the first round indirect inputs into the direct inputs you need n^2 multiplications and for each succeeding iteration, you also need n^2 multiplications.

Another method of reducing intermediate demands is by substituting, wherever possible, inputs of non-deficit materials for deficit materials. To the extent that such a substitution can take place without reducing the quality of the finished product, this method is quite satisfactory. All too often, however, one suspects that quality is sacrificed for quantity in the substitution process. Even though physical targets may then be attained, the hidden repercussions both on the quality of present products and on the need for a more rapid replacement of inferior materials in future plans must not be overlooked.

Finally, when neither of the above two procedures is successful, then the 'priority principle' of Soviet planning comes into effect. The priority principle is based on the political decisions of Soviet leaders. It consists in listing the sectors of the economy according to the degree of importance attached by planners to the attainment by these industries of their planned supplies. The main emphasis is on guaranteeing the supply of the high priority sectors. It is the sectors of secondary importance (usually the consumer oriented sectors) which usually have their allocations cut when a choice has to be made. In fact, prior to the reorganization of 1956, most of the 'belt tightening' that occurred in the Soviet Union was carried out in the household sector of the economy. However, since 1956, production of consumer goods appears to be receiving increasingly higher priorities.

SOME OPERATIONAL PROBLEMS IN CENTRALIZED SOVIET PLANNING

Since the above presentation was necessarily of a sketchy nature, the following remarks are in no way intended to represent a full, critical appraisal of Soviet planning techniques *per se*. Rather, these brief comments are meant to underline some of the more important planning problems faced by any command economy. We draw on Soviet experience merely because the economy of the USSR usually serves as the model of collectivist planning. Three major problems come immediately to mind:

1. *Optimal Degree of Centralization* 'The chief, and most persistent systematic problem of any command economy is the discovery of the optimal degree of centralization (or decentralization) under given institutional conditions and with reference to a given set of social goals.[19] By an 'optimal' degree of centralization we mean the extent to which all economic decisions should evolve from the top of the planning hierarchy so as to cause the economy to perform most efficiently within the planning framework. While by definition a command economy entails the centralization of major economic decisions concerning output, investment, and the future directions which the economy is to take, the question whether central planners should also dictate intermediate input

[19] See Gregory Grossman, 'Notes for a Theory of the Command Economy' (*Soviet Studies*, Vol. XV, October 1963), for a full treatment of this point.

flows down to the minutest detail is at the core of the planning problem in the Soviet Union. Too much centralization frequently results in a blatant inability of the Soviet system to get materials to consuming enterprises in the required quantity, or the required quality, and at the required time in the cheapest way possible. Excessive centralization also frequently leads to a lack of co-ordination between different aspects of the plan with resultant operational failures.

2. *Supply Shortages* The Soviet economy since its early days has been marked by a chronic 'seller's market', i.e. the situation where demands for material and primary inputs are continually outrunning the available supply. These chronic supply shortages are often manifested on the enterprise level by the 'hoarding' of excessive supplies, the 'padding' of material input orders, and the existence of a large quantity of unusable inventory stocks.[20] Rather than finding the surplus stocks in the hands of distributors who might put them to good use one often learns that these large inventories are of little help because they tend to be of the wrong things and in the wrong places.[21] Moreover, the ultimate cause of these supply shortages seems to be attributable to the faulty and often incomplete and inconsistent material balances worked out in the crucial third stage of the planning process.[22] The Soviet iterative method of calculating material balances is often not carried out beyond the second iteration and, thus, fails to cover enough of the *indirect* effects of a change in a number of output targets.

3. *Lack of Efficiency* The overwhelming concern of comprehensive centralized planning in the Soviet Union is simply to equate both sides of each material balance by whatever procedure seems to be most expeditious. It is often conducted with little reference to systematic and rational rules of choice or any definite notions of economic efficiency and optimality. Attention is paid more to the achievement of 'balance' than to any search for efficiency. The excessive administrative and co-ordinative burden that is placed on central planners and the necessary haste with which decisions must be made if the annual plan is to be drawn up within the specified time limits, effectively precludes all attempts at optimality in resource and material allocations. However, to the extent that a material balance can be achieved as accurately as time permits within the context of the overall planning targets, the criteria of efficiency and optimality do not appear to loom so important as in the case of market economies.

[20] Since the major success indicator in a collectivist economy is the achievement of output targets, it is not surprising to discover that Soviet managers tend to order more material inputs than they actually need in order to ensure a better chance of actually achieving required outputs. However, the role of profits is gradually being investigated by Soviet economists as a possible remedy for this chronic problem. See George R. Feiwel's book listed in Suggested Readings at the end of this chapter.

[21] Grossman, op. cit., p. 109.

[22] Levine, op. cit., p. 167.

Appendix 3.A

INPUT-OUTPUT TABLES AND THE ECONOMIES OF THE WORLD: A BRIEF SURVEY

At the beginning of chapter 2 we stated that the 'thought processes' of central planning could be illustrated best with the aid of input-output analysis. The reader was cautioned however, not to conclude from this methodological similarity that input-output is in any way the exclusive property of centrally planned economies. It is our purpose in this Appendix to give the student some idea of the extent to which the techniques of interindustry economics have been appropriated by many nations throughout the world.

Since the initial formulation of input-output analysis in the United States late in the 1930's hardly a single area of economic inquiry has been able to escape the hungry clutches of input-output enthusiasts. No less than forty-five nations now possess input-output tables in varying degrees of complexity and sophistication. These tables can be found in both communist and capitalist countries. They have been applied to fully developed as well as to less developed economies, to regional as well as to international trade. In this chapter we have seen how the input-output table might be utilized in an advanced, centrally planned economy. In a 'mixed' capitalist economy like that of the United States and the United Kingdom or in 'looser' planned economies like those of France and Italy, input-output methodology has served as a valuable aid to private businesses, e.g. for inventory and sales forecasts, as well as to the central government, e.g. for analysing foreign trade patterns and the impact of public expenditures on employment and output. In developing African, Asian, and Latin American countries, input-output tables can contribute significantly to the overall formulation of development plans as well as to specific project analysis regardless of the nature of the economic and political ideology.

But, rather than to enter upon a lengthy discussion of the historical proliferation of input-output tables and the particular uses to which they have been put in different countries, we have tried to summarize as concisely as possible this basic information in the following table. It will be noted that only one representative input-output table from various selected countries has been included along with information concerning the number of sectors into which each table is divided and the primary uses to which it has been put.[1] Due to difficulties of obtaining information, no specific details can be given about the nature of existing input-output tables in countries of the communist bloc

[1] Since the accuracy of input-output projections and plans is predicated upon the continued constancy of all input coefficients it is obvious that the usefulness of any given table is limited primarily to the short or medium period. Consequently, tables need to be revised every few years to adjust for structural changes in the economy. However, for our present purpose of merely surveying input-output tables of the world's economies we shall limit the illustration to one table per country.

although it is fairly certain that these tables do in fact exist in certain Eastern European nations (e.g. Poland and Yugoslavia). Finally, we may note that at present input-output tables for sub-saharan African nations are limited to a slightly modified version of the type presented in the last two chapters. Ghana, Zambia and Kenya are cases in point. However, as the structure of these and other newly emerging African economies becomes industrially more complex and local interindustry transactions begin to replace material and capital intermediate imports, the value of possessing a simple but statistically accurate input-output table will become ever more apparent.

Table 3.A1

A Survey of Earlier Input-Output Tables of Selected World Economies

	Country	*Date of table*	*Number of sectors*	*Primary applications*
(1)	Australia	1953	120	Analysis of foreign trade and its influence on the Australian economy
(2)	Canada	1949	42	National accounts and government statistics improvement
(3)	Japan	1955	100	Long term planning projections; analysis of structural change; import and labour requirements
(4)	Netherlands	1953	27	Policy formulation and central planning
(5)	United Kingdom	1950	10	Import content of final demand; implication of changes in final demand on total output
(6)	Puerto Rico	1956	31	Effects of economic development on structure of economy
(7)	Colombia	1953	18	Development planning
(8)	Argentina	1958	23	Development planning
(9)	Peru	1955	20	Development planning
(10)	India	1953–54	36	Development planning
(11)	France	1951	37	Structural analysis and changes in foreign trade; government finance
(12)	United States	1964	86	Government finance; private business projections; defence mobilization; foreign trade
(13)	Algeria	1954	27	Development planning
(14)	United Arab Republic	1960	33	Development planning: resource mobilization
(15)	Mali	1959	8	Development planning
(16)	Ghana	1961	14	Output and government finance projections and development planning
(17)	Northern Rhodesia (Zambia)	1961	12	Projections of influence of copper export earnings on rest of economy
(18)	Morocco	1958	30	Development planning
(19)	Tunisia	1957	27	Development planning

Sources: Chenery and Clark, *Interindustry Economics* (New York, John Wiley and Sons, 1959), pp. 184–185.
United Nations, *Economic Bulletin for Africa* (Addis Ababa, Ethiopia, June 1962), Vol. II No. 2, p. 139.

Suggested Reading

Robert W. Campbell, *Soviet Economic Power: Its Organisation, Growth and Challenge*, Second Edition (Boston, Houghton Mifflin Company, 1966).

George R. Feiwel, (ed.), *New Currents in Soviet-Type Economies: A Reader* (Scranton, Pa., International Textbook Company, 1968).

M. C. Kaser, (ed.), *Economic Development for Eastern Europe* (New York, St. Martins Press, 1968).

Herbert S. Levine, 'The Centralized Planning of Supply in the Soviet Union', in F. D. Holzman, (ed.), *Readings on the Soviet Economy* (Chicago, Rand McNally and Co., 1962).

Harry G. Shaffer, *The Soviet Economy: A Collection of Western and Soviet Views*, Second Edition (New York, Appleton-Century-Crofts, 1959).

Myron Sharpe, *Planning, Profit and Incentives in the USSR* (New York, International Arts and Science Press, 1969).

Ota Sik, *Plan and Market Under Socialism* (New York, International Arts and Science Press, 1969).

CHAPTER 4

Generalized Development Planning: AGGREGATE AND MAIN-SECTOR MODELS

INTRODUCTION

In the two preceding chapters we have tried to outline the general nature of the planning problem in an economically advanced command economy. With the aid of the static input-output model we were able to construct an analytic framework which, in our opinion, best illustrates the complicated 'thought processes' involved in drawing up a comprehensive and internally consistent central economic plan. We turn now to consider the planning problem in less developed economies where the role of theoretical economic models is considerably more direct. In most poor countries these models often form either the analytic basis or the actual operational framework within which the detailed components of the actual development plan derive their specific quantitative values. The reason for their relative importance in development planning is that the creation of these economic models adapted to the particular circumstances of an emerging economy offers, among many other attributes, the following dual advantage. First, the planning model provides a precise and systematic method of attacking specific development problems, revealing the hidden implications of seemingly unrelated individual projects, and ascertaining the resource and finance requirements of alternative economic targets. Second, a detailed development plan can greatly aid in overcoming the inconsistencies and inner contradictions that often lead to the emergence of unforeseen bottlenecks and the resultant curtailment of rapid material progress. Consequently, the following two chapters will focus on a number of basic theoretical planning models that have been employed in developing economies under various circumstances and in accordance with different objectives.

The presentation will commence with a brief analysis of some of the important considerations that need to be taken into account before choosing any particular development model or combination of models. We shall then proceed to examine three basic types of planning models. For the remainder of chapter 4 we will deal with a number of simple 'aggregate' and 'main-sector' models ranging from a slight variant of the Harrod-Domar aggregate growth model to the somewhat more complicated sectoral econometric projection models. The two distinguishing characteristics of these models which make them so amenable to development planning, especially in Africa, are first, the relative simplicity of their construction and the ease of their mathematical manipulation, and second, their adaptability to the often crude and incomplete statistical information available in most poor countries. Their main drawbacks, however, lie in the rather sweeping nature of their economic generalizations, their failure to provide a detailed structural breakdown of the economy, and their inability to ascertain whether or not a particular programme will result in an *optimal* allocation of available material, financial and human resources. To overcome these deficiencies, 'interindustry' models, mainly in the form of the 'dynamic input-output' and the combined input-output linear programming framework known as 'activity analysis', have often been adapted to the particular structural features of developing economies which are already well along the path of industrialization. Consequently, in chapter 5 we will analyse the more sophisticated and comprehensive programming techniques of these interindustry models in an attempt to understand such important notions in the theory of development planning as the simultaneous attainment of consistency, feasibility, and optimality in the overall plan.

Before commencing our discussions, however, a brief word of warning seems necessary at this point. The student is to be reminded that we will be dealing in these two chapters with only one aspect of the so-called 'economics of underdevelopment', namely, the role of theoretical economic models in formulating development plans. But, since it is beyond the scope of this book to analyse in any detail the many important social, cultural, pyschological, and political considerations that must be evaluated when undertaking the gradual but complete structural transformation of any traditional subsistence economy, we can merely call the student's attention to these 'non-economic' factors and leave it specifically to the appropriate works to deal with these issues.[1]

SOME IMPORTANT CONSIDERATIONS IN CHOOSING PARTICULAR PLANNING MODELS

As was indicated in the introduction to chapter 1, there are a number of different types of economic models and planning approaches from which a developing

[1]For example, see B.F. Hozelitz, 'Non-Economic Factors in Economic Development' (*American Economic Review*, May 1957); United Nations, *Social Implications of Industrialization*

nation can choose. A country about to commence its development programmme will have to decide which method, or combination of methods, is most suitable to its own special needs and objectives. The ultimate choice, however, is not a matter of drawing any plan out of a hat or hastily taking that approach which seems to have been most successful in some other country. Rather, an intelligent and informed choice will depend upon the answers to a number of important considerations which must be specifically analysed before reaching a final decision. Some of the more relevant considerations follow.

STAGE OF DEVELOPMENT

The choice of a particular programme obviously hinges upon the existing stage of economic maturity a country has attained. If the economy is still permeated by small scale subsistence agriculture, a limited monetary sector, and little or no interindustry relations, then detailed programming can have little applicability. It would probably be more appropriate to concentrate on individual 'social overhead' public investment projects aimed at creating the necessary conditions to enable the economy to commence its economic transformation. Some general ideas of possible overall rates of growth of GDP and its major components, as well as considerations of population growth, and in the case of most African economies, possibilities of export expansion loom more important at this early stage. In the later stages, however, the likely path of development is often more clearly discernible, and more diversified action distributed over a wider range of sectors and involving more detailed programming methods will become a greater necessity.

INSTITUTIONAL STRUCTURE

Another important consideration concerns the institutional structure of the economy and the relative roles envisaged for the public and private sectors in the development process. Where the private sector is not very influential and is expected to play a relatively passive role, the public sector will ordinarily be expected to take up the slack and provide the initial stimulus and continued overall direction. Accordingly, more attention will be devoted to public investment projects and sources of government finance. However, if the private sector is considerably more active, then the plan is more likely to concentrate on the creation of favourable conditions in which private economic activity is free to flourish. A corollary of this public-versus-private consideration concerns the general attitude and willingness of the people to co-operate with the central government in a joint effort to seek economic and social advancement, e.g. political stability is obviously a very important non-economic consideration.

in Africa South of the Sahara (New York, UNESCO, 1964); and Albert Waterston, *Development Planning: Lessons of Experience* (Baltimore, Johns Hopkins Press, 1967).

AVAILABILITY AND QUALITY OF STATISTICAL INFORMATION

The availability and reliability of statistical information necessary to initiate and formulate development plans represents a third important influence on the choice of particular models. To the extent that existing data are poor and unreliable, there will be considerably less scope for the more refined type of analysis required by some mathematical programming models. However, while the complete absence of certain statistical information may preclude the use of particular models, economic planners are often called upon to resort to 'educated guesses' or to the adaptation of empirical information from economies in similar circumstances rather than to completely forego a particularly useful approach merely for lack of complete statistical data.

OPERATIVE CONSTRAINTS

The character of the development planning model is often greatly influenced by the particular constraints or bottlenecks most operative in the economy. The most important operative constraints are usually directly related to the stage of development. However, in general, capital and foreign exchange scarcity often emerge as the principal bottlenecks limiting rapid economic development. If capital constitutes the crucial constraint, then every care must be taken to ensure its most effective and productive utilization. Labour intensive investment projects may have to be stressed so that limited capital funds can be distributed into diverse channels in the economy. When depleted foreign exchange reserves emerge as the operative constraint, export promotion and/or some form of import control will assume increased importance in the development plan. Other possible bottlenecks that might appear during the course of development include limited supplies of high-level manpower, inadequate transport facilities, and limited government finance. Lack of sufficient housing projects to accommodate the growing rural-urban migration, a common feature of the process of economic development, can also emerge as an important constraint with a number of serious non-economic implications.

PRIORITIES AND OBJECTIVES

Finally, the specific objectives which the less developed nation deems most important to the achievement of its long range economic goals must also be taken into consideration. Those most commonly accepted include the following:

(a) a rapid increase in *per capita* income,
(b) a high level of employment,
(c) a relatively stable price level,
(d) a reduction in income inequalities,
(e) a favourable balance of payments situation, and
(f) a diversified and independent economy.

While each of the above objectives may be desirable in itself, it is easy to imagine the possibility of serious conflicts developing if all are pursued with equal intensity. Therefore, it often becomes necessary to determine, in the light of existing economic conditions and constraints, the specific objective, or combination of objectives, which should receive special priority in the development plan. The remaining targets might then constitute some form of associated 'side conditions' or secondary priorities to be realized as far as possible in the course of seeking fulfilment of the main objective.

With the above considerations well in mind, we can now turn to see how a developing country might proceed to draw up an economic plan in the light of its own particular circumstances. Three basic theoretical approaches are available and will be presented in order of increasing complexity and comprehensiveness, namely the 'aggregate', the 'sectoral', and the 'interindustry' models of development programming. However, as the student will presently discover, these three degrees or 'stages' of development planning are by no means mutually exclusive. Together, they represent a logical and systematic procedure for organizing and co-ordinating the activities of various economic entities into an internally consistent and operationally effective development plan.

AGGREGATE MODELS: THE HARROD-DOMAR APPROACH

The first step in drawing up a development plan usually consists in the assessment of some maximum 'aggregate' rate of growth that the economy can be expected to accomplish given the prevailing economic forces operating at the present time and without any significant external interference. Since rates of economic growth and development are commonly expressed in terms of such aggregate quantities as gross domestic product, consumption, investment, savings, population, and employment, the overall rate of development can be most easily ascertained by analysis of past and present relationships among these principal variables. The task of the development planners will then be to determine whether or not this 'natural' or independent rate of economic growth is desirable and conducive to the fulfilment of the priorities set for the planning period. Since, in most instances, the natural maximum will be well below the desirable rate, the initial planning objective usually represents a concentrated attempt to accelerate this aggregate growth rate by designing policies to influence or control in a predetermined way the specific functional relationships revealed by the technical parameters of the aggregate model.

THE NATURAL MAXIMUM RATE OF DEVELOPMENT

The natural rate of economic development is in most instances greatly influenced by the scarcity of one or two productive resources of strategic importance. In

developing African nations the shortage of capital in one form or another can usually be singled out as the primary operative economic constraint affecting overall rates of growth. Consequently, in countries where capital is felt to represent the main bottleneck, the aggregate development model chosen most commonly utilizes some variant of the relationships embodied in the well known Harrod-Domar model. The model is greatly simplified in the sense that savings, investment and income are the only variables considered, the implicit assumption being that limited supplies of high-level manpower, fluctuations in the value of foreign trade, etc., are less crucial and that capital accumulation constitutes the central process by which all other aspects of development are made possible. The primary purpose of using this simplified model as a first step in the planning process is to arrive at some estimate of the theoretical maximum achievable growth rate of national income given the existing values of the parameter s, the rate of saving in the economy, and the parameter k, the overall or 'global' capital/output ratio.

In essence, the Harrod-Domar theory states that the rise in gross domestic product from period t to period t+1 is determined by the net amount of new capital formation in period t and its 'productivity' as measured by the inverse 'capital/output' ratio. Net investment, I_n, is assumed to be limited by the level of savings, S, which in turn is functionally related to the level of income, Y, via a propensity to save, s. The theory also assumes a stable incremental capital/output ratio, k, whose inverse 1/k, as noted above, shows the proportion of any level of net investment in period t that will emerge as increased output in t+1. The basic model can be summarized as follows:

HARROD-DOMAR AGGREGATE GROWTH MODEL

(4.1) $S = sY$, savings function, where s=average propensity to save,

(4.2) $K = kY$, capital stock-income relationship, where k=incremental capital/output ratio, and

(4.2a) $\Delta K = I_n = k\Delta Y$ where I_n=net investment.

It is assumed that

(4.3) $S = I_n = \Delta K$.

Therefore, substituting (4.1) and (4.2a) into (4.3) we get:

$$S = sY = k\Delta Y = \Delta K$$

or,

(4.4) $sY = k\Delta Y$

which yields the final expression of the Harrod-Domar relationship:

(4.5) $$\frac{\Delta Y}{Y} = \frac{s}{k}.$$

Given the statistically computed values of s and k and assuming their constancy, the Harrod-Domar equation (4.5) indicates that the maximum attainable rate of growth of GDP in the economy can be generally expressed by the ratio of s/k. For example, if a less developed country is saving approximately 6 per cent of its national income and the 'global' capital/output ratio is calculated to be equal to 4.0, then equation (4.5) states that if these parameters remain constant, i.e. there is no change in economic conditions, then economic planners can anticipate no more than a 1.5 per cent $\left(=\frac{.06}{4}\right)$ rate of income growth.

If we next introduce a variable expressing the rate of population growth, $\Delta P/P$, we can arrive at an estimate of the probable growth path of *per capita* income—a much more decisive measure of economic welfare and development. For example, if government statistics reveal that the population is growing at an average rate of 1.5 per cent per year, then given the parameters $s=0.06$ and $k=4$, we see that with no change in economic policies the growth rate of *per capita* income (calculated as $\Delta Y/Y-\Delta P/P$) will be nil. Had s been equal to 0.12, then *per capita* income could be expected to grow by 1.5 per cent since:

$$\Delta Y/Y=s/k=.12/4=.03$$

and rate of growth of *per capita* income

$$=\Delta Y/Y-\Delta P/P=.03-.015=1.5 \text{ per cent.}$$

THE DESIRED RATE OF DEVELOPMENT: SETTING AGGREGATE TARGETS

If projections of current trends in the natural rate of development indicate, as in the above example, no significant improvement in the standard of living, then such a growth rate obviously cannot be accepted as a foundation upon which to construct the development plan. It therefore becomes necessary to take measures to accelerate the growth rate of GDP beyond the natural rate in order to proceed along the desired path of development indicated by the planned targets for GDP, consumption, and investment. In terms of equation (4.5), if the natural maximum is less than the target rate of growth, the government might then take measures to either raise s (e.g. by increased taxation), lower k (e.g. by improved methods of production and/or changing the composition of investment projects), or influence both so as to increase the value of s/k and thus stimulate a more rapid rate of progress.

An alternative application of the Harrod-Domar model for setting aggregate targets, especially investment, would be to start with some desired rate of growth and calculate the required amount of investment (and thus saving) necessary to achieve the target. For example, if population is anticipated to continue growing at 2 per cent per year and planners would like to achieve a steady 3 per cent rise in *per capita* income, then it is obvious that GDP must grow at a rate of 5 per

cent per annum. Using equation (4.5) and cross multiplying, we discover that with a capital/output ratio of, say, 3.5, the rate of savings will have to rise to 17.5 per cent of GDP in order to ensure that the necessary finance will be available. This result can be obtained by using the following variation of the basic Harrod-Domar formula:

$$\frac{I_n}{Y} = \frac{S}{Y} = k(W_p + P) \tag{4.6}$$

where I_n/Y is the rate of investment,

$\frac{S}{Y}$ is the rate of domestic savings,

k is the capital/output ratio,

W_p is the desired rate of growth of *per capita* income,

and

P is the statistically estimated rate of population growth.

Thus, since $k=3.5$, $W_p=3.0$ per cent and $P=2.0$ per cent, we have from equation (4.6) $I_n/Y=S/Y=3.5\ (3.0+2.0)=17.5$ per cent.

The three main sources of domestic saving, S, are: (1) personal savings, (2) business savings, and (3) government savings. If it turns out that personal and business savings do not amount to the required 17.5 per cent of GDP, then the government must raise tax rates in order to restrict consumption and thereby fill in the financial gap between actual savings and desired investment. Alternatively, if private savings plus the public surplus are still insufficient to finance the required rate of investment, then the government is in general faced with two alternative courses of action. It can seek foreign assistance in the form of grants and loans, or simply make the plan less ambitious by a downward readjustment of its target rate of growth. In most African nations, however, where the foreign sector plays such a crucial role in development planning, there is still a third major alternative consisting of a concentrated attempt to stimulate exports, curtail non-essential imports, and promote import-substituting industries with the intention of improving the balance of trade and obtaining more foreign exchange. Whatever the case may be, however, the aggregate model provides a quick overall look at the prevailing economic situation which, though greatly simplified and restricted, can offer a rough approximation of general trends and provide a basis for a more detailed analysis and breakdown of the principal variables included within its framework.

COMPLETE MAIN-SECTOR MODELS

While the setting of general target rates of economic development is a useful starting point, economic planning, even in the most backward nations, usually involves at least a further breakdown of the plan to cover the broad sectors of

the economy.[2] Target rates of growth are then computed for each of these different sectors with an eye towards the gradual structural transformation of the country's economic base. For example, one of the major objectives of economic development is to improve the productivity of the agricultural sector in order to provide necessary manpower and food supplies to support the creation of a self-sufficient industrial complex as well as to expand export earnings. Already, therefore, one can see that we have distinguished among three sectors of the economy, the domestic-oriented agricultural sector, the industrial sector, and the export sector. In order to plan the development of these main sectors, however, aggregate data must be broken down into relevant sectoral components. As long as the number of sectors does not grow too large and detailed, the techniques of sectoral planning remain relatively simple and within the statistical capabilities of most planning agencies in developing countries.

CHARACTERISTICS OF MAIN-SECTOR MODELS

In order to be used as a framework for the formulation of an overall development plan, sectoral models must exhibit the following four properties:

1. The sectors that comprise the model should cover the whole economy, i.e. the model must be *complete*. This condition does not imply that the model must consist of a great number of sectors in order to have complete coverage. But it does imply that all aspects of economic activity be accounted for either explicitly or implicitly in one of the sectors of the model. For example, it is not sufficient to construct a two-sector model consisting of an 'agricultural' sector and 'industrial' sector unless it is previously stated that such activities as transport, trade, and finance are implicitly submerged and grouped together under the category 'industry'. For such cases, it would make the model more complete if the two sectors were designated 'agriculture' and 'non-agriculture'.
2. The model should be *economical* in the sense that it concentrates primarily on the most important sectors of the economy and does not devote too much attention to minor activities.
3. The model should be *realistic* in the sense that functional relationships are truly representative of the prevailing structural conditions in the economy and estimates of the values of the principal variables and strategic parameters are based on accurate accounting procedures and adequate statistical series.
4. Finally, as far as possible, the model should be *internally consistent* and free from inner contradictions. For example, the combined output targets for

[2] Although the term 'sectoral model' is often used to describe a model depicting the activities of one particular sector, say agriculture, the following discussion will be concerned solely with those models which view the economy as being composed of a number of *main sectors* which, when taken together, cover all productive activities. (See the introduction to chapter 1 for this distinction).

agriculture and non-agriculture should not require more productive or financial resources than can be expected to be available. Checks for internal consistency usually involve more advanced programming techniques of the type to be discussed in the next chapter. Nevertheless, crude calculations can still be made at this level of planning based on such concepts as sectoral capital/output and labour/output ratios.

SOME EXAMPLES OF SIMPLE TWO-SECTOR MODELS[3]

Consumption- versus Investment-Goods Sectors

Suppose the planning authorities of, say Kenya, have decided to concentrate their attention on the supply of consumption-goods and investment-goods in the next five-year plan to make certain that the production of these two basic types of goods is in accordance with some desired general rate of development. Consequently, for purposes of analysis, the economy is to be divided into two sectors: the consumption-goods sector and the investment-goods sector[4] where it is assumed that there are no intersectoral intermediate transactions. The sum of the outputs of both sectors must, therefore, represent the level of gross domestic product in Kenya. Denoting the output of sector 1 (the consumption-goods sector) as X_1, and sector 2 (investment-goods) as X_2, we have:

(4.7) $\quad X_1+X_2=\text{GDP}.$

Given the marginal propensity to consume, b, which for simplicity is here assumed to be equal to the average propensity, the supply of consumption-goods must be sufficient to satisfy the demand for these goods where the level of total demand is given by the following consumption function:

(4.8) $\quad X_1=b(X_1+X_2).$

We see from equation (4.8) that output, or productive capacity, in the two sectors must bear a constant relationship to each other as determined by the propensity to consume. Rearranging the terms of equation (4.8) and dividing we see that this relationship is given by:

(4.8a) $\quad \dfrac{X_1}{X_2}=\dfrac{b}{1-b}.$

For example, if $b=0.75$, then the relative ratio of consumption-goods output

[3] The following discussion and illustrations are based on the material of, United Nations, ECAFE, *Programming Techniques for Economic Development* (Bangkok, 1960), chapter III.

[4] To ensure the completeness of this model, the consumption-goods sector is assumed to cover the production of all non-investment type goods and services (e.g. basic foodstuffs, clothing, personal items, etc.). Obviously, in actual practice, we would want a further breakdown of these two sectors into their own subsectors. However, the analytic approach would still remain essentially the same.

to investment-goods output must continuously remain constant at 3 to 1 $\left(\text{i.e. } \frac{X_1}{X_2}=\frac{0.75}{0.25}=\frac{3}{1}\right)$. This implies that the two sectors must grow at the same relative rates. However, if the propensity to consume is expected or is planned to gradually fall as national income rises (e.g. to provide more savings for larger investment endeavours), then investment-goods production must begin to grow at a relatively accelerated rate. In fact, in the early stages of development planning one would normally expect the government to make a determined effort to alter the existing relationship in favour of investment-type goods, i.e. to lower $\frac{b}{1-b}$.

One further step is needed to complete this two-sector model. We must recognize that part of the investment-goods output must be devoted to expanding the productive capacity of the consumption-goods sector. Unless more productive capacity is created in this sector, future increases in consumption demand will not be satisfied. Given the desired rate of income growth, the demand for consumption-goods in any period will be determined by equation (4.7). If this level of demand exceeds the limits of existing productive capacity of the consumption-goods sector, then the resultant supply shortage will have to be ameliorated by further investment with the aim of creating more capacity. The machines and equipment necessary to expand production will have to come from the investment-goods sector. For example, suppose the present capacity of the consumption-goods sector is 300 and it is to be increased by 5 per cent per year. This means that if the consumption-goods sector is presently operating at or near full capacity and if its incremental capital/output ratio k, is say, 4.0, then in order to provide the increased output capacity of 15 (i.e. $.05\times 300=15$) for the next period, investment *in the present period* must be $4\times 15=60$. But these 60 units must be produced by the investment-goods sector whose capacity therefore must necessarily exceed this amount. Suppose the investment sector had a capacity of 100. If it produces 60 for the consumption sector in this period it will have an excess capacity of 40 which can be used to expand its own output. Thus, if the capital/output ratio of the investment sector k_2 is say, 8.0, then these 40 units could be utilized to expand its own capacity in the next period by $40/8=5$ units of output. In terms of our model, the output of the investment-goods sector will be given by the following relationship:

(4.9) $X_2=k_1\,\Delta X_1+k_2.\,\Delta X_2$

where,

k_1 is the capital/output ratio of sector 1, and
k_2 is the capital/output ratio of sector 2.

It is evident from the above discussion and from equations (4.7) to (4.9) that for our hypothetical two-sector model to be both complete, internally consistent

and capable of projecting output levels over a number of periods, we seem to need the following four pieces of information:

1. a target rate of growth of national income, $(\Delta X_1+\Delta X_2)/(X_1+X_2)$;
2. the base year levels of productive capacity in each sector, X_1^o and X_2^o;
3. the propensity to consume, b; and
4. the incremental capital/output ratios of the two sectors, k_1 and k_2.

In reality, the first piece of information is superfluous since the general rate of economic growth can be calculated within the model itself using the Harrod-Domar approach described in the preceding section. For example, if we know the propensity to consume, b (and it is expected to remain constant), then the propensity to save, s, will also be constant and equal to 1–b. Similarly we can compute the global capital/output ratio, k, for the entire economy as the weighted average of our two sectoral capital/output ratios since the model is by definition complete in its coverage. Thus, in the above example, we know that the ratio of production in the two sectors remains constant at 3 units of consumption-goods output for every one unit of investment-goods production. Since their capital/output ratios are 4.0 and 8.0 respectively, the global capital/output ratio can be calculated as the following weighted average:

$$k=b\ k_1+(1-b)\ k_2$$

or, in our numerical example:

$$k=0.75\times 4.0+0.25\times 8.0=5.0.$$

Similarly, we can rewrite equation (4.5) of the Harrod-Domar aggregate model (i.e. $\Delta Y/Y=s/k$) in terms of the present two-sector model by substituting the above weighted average expression into the denominator to obtain the following formula:

$$\text{rate of growth of GDP, } \frac{\Delta X_1+\Delta X_2}{X_1+X_2}=\frac{s}{k}=\frac{1-b}{bk_1+(1-b)k_2}.$$

Consequently, we find that with a propensity to consume of 0.75 and a combined capital/output ratio of 5.0, our maximum attainable rate of income growth will be equal to 0.25/5.0, or five per cent per year.

The internal consistency of this simplified two-sector model can be usefully demonstrated by the following tables (4.1) which are based on the three equations of the model and the above numerical values for base year GDP and for the parameters b, k_1, and k_2.

In an analogous but slightly more mathematically involved procedure we could equally as well calculate future output relations between these two sectors on the basis of disproportionate relative rates of growth by no longer requiring our three strategic parameters to remain constant over the planning period.

Export versus Home-Market Sectors

In most developing economies where foreign trade represents a sizeable proportion of all economic activity, a two-sector model consisting of export versus

Table 4.1
A Simplified Two-Sector Model: Consumption and Investment
Period One (Base Year)

↓ *Payments to* *Outputs from* →	Consumption	*Investment* Consumption-goods Sector	Investment-goods Sector	*Total Product*
Consumption-goods Sector	$0.75\times400=300$			300
Investment-goods Sector		$4.0\times15=60$	$8.0\times5=40$	100
Income Payment	300	100		400

Period Two

↓ *Payments to* *Outputs from* →	Consumption	*Investment* Consumption-goods Sector	Investment-goods Sector	*Total Product*
Consumption-goods Sector	$0.75\times420=315$			315
Investment-goods Sector		$4.0\times15.75=63$	$8.0\times5.25=42$	105
Income Payment	315	105		420

As long as the propensity to consume and the capital/output ratios remain constant we can generalize the solution to this model as follows:

Period N

↓ *Payments to* *Outputs from* →	Consumption	*Investment* Consumption-goods Sector	Investment-goods Sector	*Total Product*
Consumption-goods Sector	0.75×400 $(1.05)^{n-1}$			Same as Col. 1
Investment-goods Sector		4.0×15 $(1.05)^{n-1}$	8.0×5 $(1.05)^{n-1}$	Col. 2+ Col. 3
Income Payment	0.75×400 $(1.05)^{n-1}$	4.0×15 $(1.05)^{n-1}$	8.0×5 $(1.05)^{n-1}$	$400\times$ $(1.05)^{n-1}$

home-market production may be more realistic and useful than the model just described. To the extent that the export sector also sells some of its output to the home-market (e.g. part of Uganda's coffee, tea and cotton output is sold locally), this two-fold division will not be strictly correct. It will be assumed in the following presentation, however, that the economy can for analytic purposes be justifiably split into two distinct sectors, one specializing in export and the other in domestic activities, and that the only intermediate transactions between the two sectors are purchases of new capital goods from the domestic market by the export sector.

The main difference between this and the previous two-sector model is that in this model the growth of one sector's output, the export sector, depends

greatly upon exogenous international economic developments while the demand for the goods of the home sector is primarily determined by local levels of consumption and investment, and can be manipulated by domestic economic policies. As such, planning for economic development in countries whose export sector is especially important becomes much more difficult due to the economy's ultimate dependence on the vicissitudes of world prices and demands for primary export products. Given this difficulty of controlling export earnings, the best starting point for planning on the basis of this model is to estimate one or more probable rates of export expansion. If prices are assumed to remain relatively constant, these projections can be based on estimates of future incomes in the developed countries and on their income elasticity of demand for our exports. Alternatively, accurate export projections are probably more realistically arrived at if, in addition to the above considerations, variable prices and price elasticities of demand are recognized as being equally influential. For example, the future demand for exports might be calculated on the assumption that primary product prices will decline at an average rate of, say, one per cent per annum based on past experience. Of course, in reality, one must recognize that many other factors such as tariff policies, growing competition from synthetic materials, and international trade agreements, can easily invalidate our econometric projections. But since many of these factors cannot readily be anticipated, planning authorities usually abstract from them when projecting future export earnings with the implicit intention of making the required adjustments whenever the necessity arises.

Since it is assumed in this model that the export sector does not sell to the home market, the former's rate of growth will obviously depend on the rate of increase in the demand for exports. In fact, these two rates must necessarily be equal. Thus, one equation of this model would equate the output of sector 2, the export sector in period t, with the total demand for export-type goods from this country in period t, that is:

(4.10) $X_2 = E\,(t)$.

The demand for exports in period t is assumed to be determined by the anticipated rate of growth of export demand, r. It is therefore calculated by the following exponential equation:

(4.11) $E\,(t) = E_o(1+r)^t$

where E_o is the total value of base year exports ($=X_2^o$). Given the export sector's capital/output ratio k_2, its demand for the investment goods produced by the domestic sector will be given by $k_2\Delta X_2$. Similarly, the home-market sector must also produce investment goods for its own capacity requirements as well as consumption goods to satisfy a domestic demand which is functionally related to the total income of *both* sectors, i.e. the total demand for home-market type goods will consist of consumption demand ($=b\,(X_1+X_2)$) plus the domestic sector's own investment demand ($=k_1\Delta X_1$) plus investment demand of the export sector ($=k_2\Delta X_2$). But since a portion of total home demand will ordina-

rily be partially satisficd by the importation of consumer and capital goods, only the remainder will be supplied by local production. Consequently, we may define the domestic supply of locally produced goods, X_1, as equal to the total demand for the products of this home-market sector minus imports, that is:

(4.12) $X_1=b(X_1+X_2)+k_1\Delta X_1+k_2\Delta X_2-M.$

Finally, it will be assumed in this model that the demand for imports, M, bears a linear relationship to the level of GDP, the factor of proportionality being a constant marginal propensity to import, m. Therefore, to complete our model we need one more equation, namely:

(4.13) $M=m\,(X_1+X_2).$

With these four relationships in mind, let us now see how this hypothetical two-sector model might be utilized, say, in a country like Zambia.[5]

Suppose that on the basis of empirical investigation of past trends in the Zambian economy the following parameters were derived:

(1) average=marginal propensity to consume, $b=0.80$;
(2) capital/output ratio of home-market sector, $k_1=3.5$;
(3) capital/output ratio of export sector, $k_2=6.0$;
(4) marginal (=average) propensity to import, $m=0.20$.

If the demand for copper exports was expected to grow at an average rate (r) of five per cent per annum and if in the base year GDP was equal to, say, 1,000, then it is evident that the overall rate of economic growth will also equal five per cent per annum, since:

$$\text{rate of growth of GDP} = \frac{\Delta X_1+\Delta X_2}{X_1+X_2} = \frac{s}{k}$$

$$= \frac{1-b}{(k_1\times 0.8)+(k_2\times 0.2)} = \frac{0.20}{(3.5\times 0.8)+(6\times 0.2)}$$

$$= 0.05.$$

In terms of our numerical example, the following set of charts in Table 4.2 provides a detailed picture of the movement of the components of this two-sector model over time.[6]

Although this illustration depicts a constant five per cent rate of growth in both sectors, the reader should notice that while the growth rate of the export sector is completely determined by the growth rate in demand for exports (r), the home market's growth is influenced not only by export growth but also primarily by the parameter b, the propensity to consume. The lower the propensity to consume, the higher will be the rate of growth in the home market since more capacity will be available for the production of investment-

[5] Since very little of Zambia's major export product, copper, is sold to the domestic market, our simple home-versus-export model seems to be best suited to the Zambian economy—at least, for pedagogic purposes.

[6] United Nations, ECAFE, *Programming Techniques for Economic Development* (Bangkok, 1960), p. 26.

Table 4.2

Export Sector versus Home-Market Sector: A Hypothetical Zambian Example

Period One (Base Year)

↓ *Payments to Outputs from* →	*Consumption*	*Investment* Home-Market Sector	*Investment* Export Sector	*Total Exports*	*Total Imports*	*Total Production*
Home-Market Sector	$0.8 \times 1{,}000 = 800$	$3.5 \times 40 = 140$	$6 \times 10 = 60$		$-(.20 \times 1{,}000) = 200$	800
Export Sector				200		200
Income Payment	800 (Home-Market Sector)			200 (Export Sector)		1,000 (GDP)

Period Two

↓ *Payments to Outputs from* →	*Consumption*	*Investment* Home-Market Sector	*Investment* Export Sector	*Total Exports*	*Total Imports*	*Total Production*
Home-Market Sector	$0.8 \times 1{,}050 = 840$	$3.5 \times 42 = 147$	$6 \times 10.5 = 63$		$-(0.20 \times 1{,}050) = 210$	840
Export Sector				210		210
Income Payment	840 (Home-Market Sector)			210 (Export Sector)		1,050 (GDP)

Period Three

↓ *Payments to Outputs from* →	*Consumption*	*Investment* Home-Market Sector	*Investment* Export Sector	*Total Exports*	*Total Imports*	*Total Production*
Home-Market Sector	$0.8 \times 1{,}102.5 = 882.0$	$3.5 \times 44.1 = 154.35$	$6 \times 11.025 = 66.15$		$-(0.2 \times 1{,}102.5) = 220.5$	882.0
Export Sector				220.5		220.5
Income Payment	882.0 (Home-Market Sector)			220.5 (Export Sector)		1,102.5 (GDP)

Period N

→ *Payments to Outputs from* →	*Consumption*	*Investment* Home-Market Sector	*Investment* Export Sector	*Total Exports*	*Total Imports*	*Total Production*
Home-Market Sector	$0.8 \times 1{,}000 \times (1.05)^{n-1}$	$3.5 \times 40 \times (1.05)^{n-1}$	$6.0 \times 10 \times (1.05)^{n-1}$		$-0.2 \times 1{,}000 \times (1.05)^{n-1}$	Same as Column 1
Export Sector				$200 \times (1.05)^{n-1}$		Same as Column 4
Income Payment	$0.8 \times 1{,}000(1.05)^{n-1}$ (Home-Market Sector)			$200(1.05)^{n-1}$ (Export Sector)		$1{,}000 \times (1.05)^{n-1}$ (GDP)

type goods. The fact that the home-market rate of growth was identical with the export rate was merely a coincidental result of the specially chosen values of the parameters b, k_1 and k_2. Suppose, for example, that in Period Two exports are expected to grow by six rather than by ten units (i.e. r falls from five to three per cent). The increase of six requires an investment of $6\times6=36$ in Period One. Assuming all internal parameters remain the same, this investment of 36 implies that home-market investment in Period One can amount to 164 which will increase the capacity of this sector by $164\div3.5=46.9$ in the second period. As a result, the home-market sector grows faster than the export sector. But we must not neglect considerations of the impact of such occurrences on the balance of payments situation of Zambia. For, if domestic production rises to 846.9 units in Period Two, imports will consequently rise to a level of 211.7 units which results in a 5.7 unit deficit in the balance on current account. Unless this unfavourable balance is counteracted by the planned development of import-substituting industries, or adequate foreign capital flows, then the higher growth rate of the domestic sector could lead to a serious drain on foreign exchange reserves which in turn could obviously have an adverse effect on domestic growth. What do you think would be the balance of payments implications, however, if in Period Two exports were expected to grow by 8 per cent? Or if the marginal propensity to consume falls to 0.75? Or if the marginal propensity to import rises to 0.25? Whatever the case may be, it is evident that this dual sector model involves three fundamental considerations:

(1) the supply and demand for home-market goods,
(2) the supply and demand for export-type goods, and
(3) the balance of payments.

CONCLUDING REMARKS: FROM TWO-SECTOR TO MULTI-SECTOR PROJECTION MODELS

Although for descriptive simplicity we have restricted the foregoing presentation to two dual-sector models each portraying the most important structural features of different developing economies, we could easily combine these models and formulate three, four, and five-sector models. The methodology of the planning procedure, however, would be essentially unchanged although specific mathematical computations obviously become more complicated as more sectors are added. For example, we might combine the two models just presented into a single three-sector model consisting of the export sector, the consumption-goods sector, and the investment-goods sector, and proceed in an analogous manner.

Alternatively, if somewhat more detailed sectoral information is available as in a number of contemporary African economies, planners often attempt to construct a 'multi-sector statistical projection model'. This type of model usually consists of five to ten main sectors which, when analysed together, are

intended to summarize in a slightly more comprehensive manner than two and three-sector models the special structural features of a particular economy. A model of this sort must be carefully fitted to existing statistical series before being used to make projections of intermediate term output and finance possibilities for the development plan. For example, Professor Paul G. Clark[7] developed a six-sector projection model for Uganda which was later fitted to Kenyan and Tanzanian data.[8] Its primary purpose was, as Professor Clark noted, first 'to provide the link between the desired objectives for the economy as a whole and the development expenditures and policies subject to government decision', and second, 'to estimate, from these given objectives, the "required" development expenditures and policies to attain them, or from given development actions, the "expected" attainment of objectives.'[9] Such a multisector projection model, if constructed on the basis of fairly reliable statistics, certainly represents a significant analytic and operational improvement over both the aggregate Harrod-Domar and the 'semi-aggregate' two and three main-sector models of this chapter. However, whenever planners seek a more detailed industry by industry framework for use in ascertaining potential sources and uses of domestic and imported material and capital goods, or whenever the development model must take account of alternative methods of production *within the same industry* or discover some optimal pattern of production for the economy as a whole, main-sector projection models must ordinarily be supplemented by the more elaborate interindustry programming models which are discussed in the next chapter.

[7] Director of the East African Institute of Social Research at Makerere University College between 1963 and 1965.

[8] See P. G. Clark, 'The Rationale and Use of a Projection Model for Uganda' (*Economic Development Research Project*, paper 39, July 1964). The six sectors consisted of agriculture, manufacturing, construction, transport, services, and government.

[9] ibid., p. 2.

Suggested Readings

P. G. Clark, 'The Rationale and Use of a Projection Model for Uganda' (*Economic Development Research Project*, paper 39, East African Institute of Social Research, July 1964).

Richard Eckhaus and Kirit Parikh, *Planning for Growth: Multisectoral, Intertemporal Models Applied to India* (Cambridge, Mass., MIT Press, 1968).

David A. Kendrick, *Programming Investment in the Process Industries: An Approach to Sectoral Planning* (Cambridge, Mass., MIT Press, 1967).

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CHAPTER 5

Comprehensive Development Planning:

DYNAMIC INPUT-OUTPUT AND ACTIVITY ANALYSIS

As we have seen in the previous chapter, a model depicting the interdependences among two or more main sectors of the economy can greatly aid in the construction of a development plan that takes account of broad intersectoral relationships. However, when it comes to the formulation of detailed and comprehensive development plans where internally consistent production targets and import requirements are desired on an industry by industry basis, each branch of the broad sectoral model must be broken down even further into its component subsectoral units and analysed within some variant of the basic input-output framework studied in chapter 2.[1]

Although interindustry models are primarily applicable in economies that have achieved a certain degree of industrial development where a considerable volume of statistically recorded intermediate material transactions takes place, the construction of a simple input-output table can, nevertheless, also make a significant contribution in countries where industrial activity is still in its infancy. Thus, although practical applications of the sort described in chapters 2 and 3 may be precluded by a lack of sufficient data in most contemporary African nations, the preparation of a rough input-output table is quite often a useful way to organize the data that do exist and to locate statistical inconsistencies and deficiencies where further investigation would be of great value. Furthermore, the existence of many 'empty boxes', i.e. zero interindustry transactions, revealed by the rough input-output table provides an excellent

[1] For example, the 'agriculture' sector might be sub-divided into individual subsectors such as 'sugar', 'cotton', 'coffee', 'sisal', 'cocoa', 'groundnuts', etc., and treated as separate entities in the input-output model.

means of estimating the total potential contribution that the planned introduction of any new industry can be expected to make on income, output, and employment in the overall economy. A related use of the input-output table is to serve as an intermediate step in the construction of a detailed system of national income accounts.[2]

SOME PRACTICAL USES OF INPUT-OUTPUT MODELS IN DEVELOPMENT PROGRAMMING

Our discussion of the input-output methodology in the earlier chapters touched upon some of the areas of central planning in fully developed command economies where the model can be of considerable practical and analytic value. In the field of development programming where the number of producing sectors is relatively small (say fifteen to, at most, forty major industries in most developing economies), the practical applicability of the input-output technique assumes a position of paramount importance. With only, say, twenty-five industries to deal with, the mathematical manipulations of the input-output model are rendered relatively simple since the computation of an inverse matrix for such a table is readily attainable with even the most simple variety of modern electronic computer. In addition, many crucial analytic areas of development planning, such as the computation of internally consistent labour, capital, and import requirements on the basis of alternative output or investment targets, can be handled with relatively unsophisticated mathematical methods. Let us, therefore, examine some of the more important practical uses of the simple input-output model in the field of development planning.

IMPORT REQUIREMENTS AND SUBSTITUTION POSSIBILITIES

While the primary application of input-output in developed centralized economies is to provide a method for arriving at detailed production levels that are consistent with each other and with final demand targets as expressed, for example, by the 'material balance' of the Soviet plan, the practical utility of input-output in developing economies, oriented as they are towards foreign trade, is derived more from its usefulness in estimating import requirements and evaluating possibilities of import substitution. Consequently, when drawing up an input-output table for a developing economy, planners often use a 'modified' form of the basic structural framework described in the beginning of chapter 2.[3] Rather than a single row for imports and a single column for exports,

[2]For example, see B. Van Arkadie and C. Frank, *Economic Accounting and Development Planning* (Nairobi, Oxford University Press, 1966), chapter VIII.

[3] For example, in chapter III of Van Arkadie and Frank, op. cit., a modified input-output table constructed by Professor Dudley Seers for use in developing countries is presented.

several rows and columns are often utilized to distinguish among various categories of imports and exports. For example, the import sector could usefully be broken down into separate rows representing (a) imports of raw materials and intermediate products, (b) imports of finished goods, (c) imports of capital goods, and (d) transfers such as loans and grants for specific projects. Furthermore, for countries like those of East Africa where a common market of considerable importance exists, there may be separate rows and columns for import and export transactions between, say, Kenya and Tanzania, and Kenya and Uganda. Such a distinction between trade with the rest of the world and other common market countries would, in the East African situation, greatly aid in evaluating the desirability of establishing specific import-substituting industrial projects in any one country and the impact that these projects might have on the economic well-being of the entire area.[4]

The more precise treatment which interindustry economics affords to the foreign trade sector can also contribute significantly to any analytic breakdown of sources and uses of foreign exchange. By distinguishing between, say, imports of intermediate materials which depend primarily on the level of total output in any particular industry, and imports of capital goods which depend on the specific levels of investment required in each industry to expand productive capacity by one unit in the next period (i.e. the industry marginal capital/output ratio), the input-output table recognizes that the importation of different types of commodities will depend on these various economic activities. Thus, the decision to create a specific new industry designed to replace import-type goods with locally produced products will be greatly affected, not only by the present structure of import activity in all other sectors of the economy, but also by the direct and indirect impact that this new industry might be expected to have on future import requirements of these same sectors. In many cases one discovers that the creation of an import-substituting industry has in fact led to an expanded volume of total imports when all indirect repercussions are taken into account.

CHOICE AMONG EXPORT-PROMOTION ALTERNATIVES

One often finds in the literature on economic development the argument that in order to overcome their dependence on the fluctuations of primary product export prices, less developed African economies might do best to embark upon a vigorous programme of export diversification and export promotion rather than to follow a single policy of import substitution. In such circumstances,

[4] Ideally in East Africa one might eventually hope to develop a 'regional' input-output table linking the activities of each individual country (here considered a region of East Africa) with those of the others. For a general discussion of regional input-output tables see Chenery and Clark, 1959, pp. 65–70.

the input-output methodology can be called upon to evaluate the impact of alternative export projects on income and employment in the domestic economy as well as on levels of anticipated foreign exchange. For example, the foreign trade sectors of the input-output table could be disaggregated into individual components reflecting exact country or currency of origin and destination of exports and imports, in order to obtain a separate 'foreign trade matrix' within which alternative possibilities of export diversification might be ascertained. Given the expected values of any new export activities, we could then analyse the impact of these new industries on output, employment, and imports of all other sectors of the economy.

LABOUR AND MANPOWER ANALYSIS: LEARNING CURVES

In our discussion of the basic input-output model, we derived an equation that could be used to express the general relationship between the level of total output in any industry and its resultant level of employment, namely, the expression $L_i = l_i X_i$. Similarly, we expressed total industrial employment, L_T, in the economy in terms of the following linear equation:

$$L_T = \sum_{i=1}^{n} l_i X_i.$$

Given the various labour coefficients, $l_{i\ i=1, 2, \ldots, n}$, and production targets, X_i, we were then able to determine individual and total levels of employment. Since the l_i's represent the labour/output ratios in each sector of the economy, it follows that the inverse of l_i, $\frac{1}{l_i}$, can be expressed as the output/labour ratio which, as we know from chapter 2, represents the 'average' productivity of labour in industry i. Now, one of the distinguishing features of an underdeveloped economy is its low level of labour productivity. The task of the development plan will often be, among other things, to improve labour's productivity, i.e. to lower the labour coefficients, l_i. But this implies that calculations of employment based on the assumption of constant labour coefficients (an inherent feature of the basic input-output model) will normally *over-estimate* the level of employment required to produce the planned output targets. Since it is more realistic to assume that over the course of development plans labour productivity can be expected to improve steadily as production goes up, economic planners have developed what are known as 'learning curves' or 'progress functions'. These functions, relating labour productivity to various levels of total output for different industries, have been empirically estimated both by 'time-series' and 'cross-section' studies of growing economies and are available for use and adaptation by economic planners in different countries. Thus, when calculating the employment requirements based on target outputs for, say, five years hence, planning agencies could, in the absence of local information, adjust present labour coefficients in accordance with these empiri-

cal 'learning curves' so as to reflect the expectations of improved productivity at the end of this five year period.

EFFICIENT INVESTMENT ALLOCATION: DYNAMIC INPUT-OUTPUT ANALYSIS

One of the main problems of applying the methodology of the 'static' input-output model to the field of development planning is that this model fails to take account of one very important phenomenon of economic development, namely, that investment cannot realistically be considered an autonomous final demand like consumption, government spending, and exports. Rather, we must recognize that the magnitude of capital accumulation in any one year is itself highly dependent on the level of consumption, government expenditure, and especially in Africa, on the value of total exports. The model analysed in chapter 2 was based on the assumption that only current flows of inputs and outputs are important aspects of the economic plan. Specifically, this model assumed that investment can be included as one of the components of an exogenously determined level of final demand for the goods of a particular industry and that its magnitude in any given year is unassociated with the level of economic activity in that industry. By so doing, the static input-output model divorces investment decisions from output objectives and capacity considerations. Since it is intuitively clear that the level of investment in any sector of a developing economy depends greatly on the degree to which existing capacity is being utilized and that limited capital goods must be allocated between industries in an efficient way, it becomes necessary to build investment requirements into the basic framework. By means of a simple refinement of the static model, therefore, development planners can 'dynamize' the input-output model by including 'subsectoral capital coefficients' which relate investment needs to continuously changing present and future output targets. The resultant model is known as the 'dynamic input-output model'.

In reality, the introduction of a matrix of subsectoral capital coefficients into the basic input-output framework represents merely a further extension of the concept of incremental capital/output ratios employed in both the aggregate and main-sector models discussed in the previous chapter. It recognizes the fact that in addition to the current intermediate material 'flow' requirements of any economic system (reflected in the matrix of technical coefficients of production), there exist capital or 'stock' requirements (described by a matrix of capital coefficients) that must also be satisfied if the necessary buildings, machinery and inventories are to be available to satisfy output objectives. Dynamic input-output analysis thus attempts to take account of capital requirements and economic growth over time. Before presenting the mathematical aspects of the dynamic model, however, let us see if we can intuitively grasp this important notion of subsectoral capital stock requirements.

As was pointed out above, any industry must purchase not only current intermediate inputs (represented by $\sum_{i=1}^{n} x_{ij}$ in our model) but also capital inputs needed to expand its productive capacity. Like most other dynamic models, the dynamic input-output model is based on the assumption that there exists a unique technological relationship in each industry between any change in the level of total output and the necessary change in the size of the capital stock required to produce this output. This relationship is expressed by the incremental capital/output ratio or, as it is sometimes called, the incremental capital coefficient. For example, suppose that the 'industrial' capital/output ratio for, say, the electricity 'industry' of Uganda is 6.0. This means basically that 6.0 units of capital goods must be invested in this period to increase the level of output capacity by one unit in the next period.[5] Since the Uganda Electricity Board must normally purchase a variety of capital goods to expand its productive capacity, it is natural to assume that these 6.0 units of capital will consist of, say, 2.5 units of steel, 1.5 units of cement, 1.7 units of machinery, and 0.3 units of timber. Thus, while the aggregate Harrod-Domar model considered only a global capital coefficient of which the electricity coefficient was merely a small part, and the sectoral models disaggregated this coefficient into sectoral capital/output ratios, we see that the dynamic input-output model attempts to disaggregate these sectoral ratios even further into 'subsectoral capital coefficients.' In the above example, the four subsectoral coefficients for Uganda's electricity industry relating the number of units of capital goods which the UEB must purchase from steel producers (whether at home or abroad), the cement industry, producers of industrial machinery, and the timber industry, in order to expand its capacity by 1 unit are 2.5, 1.5, 1.7, and 0.3 respectively. Clearly, these coefficients must add up exactly to the sectoral capital/output ratio of electricity industry, namely, 6.0. Once we know these subsectoral ratios, it becomes an easy task to estimate the amount and type of capital goods that must be purchased in this period if output in the next period is planned or expected to rise by any given amount. Suppose, for example, that statistical projections of the expected demand for electricity in Uganda in year t+1 indicate that the UEB will have to expand by approximately 100 units beyond the productive capacity of period t.[6] Given the above capital coefficients, in order to attain

[5] This one-period lag between capital formation and additional capacity gives the model its 'dynamic' properties.

[6] Recall from chapter 2 that to estimate the total output requirements of the UEB designated as say, industry 3, we would first have to know the total demand not only for electricity but for all other products produced in Uganda and the matrix of technical coefficients and then apply the formula

$$X_3 = \sum_{j=1}^{n} r_{3j} Y_j$$

where r_{3j} represents the elements in the third row of the inverted (I−A) matrix.

this additional productive capacity in $t+1$, the UEB will have to invest in 250 units of steel (i.e. 2.5×100), 150 units of cement, 170 units of industrial machinery, and 30 units of timber during period t. A similar procedure could be employed to estimate the capital needs of all other industrial sectors of the economy. Therefore, we must adjust our static input-output model to account for these dynamic interindustry capital requirements so that we may arrive at a final system of input-output equations that might be particularly useful for medium range development programming.

THE DYNAMIC INPUT-OUTPUT MODEL[7]

Since the student is already acquainted with the mathematics of the static input-output model, we will use the symbols of that framework to summarize the dynamic model. Suppose we let $V_{ij}(t)$ represent the stock of commodity i, say cement, produced by industry i and purchased *as capital goods* by industry j, say the textile industry, at time t. The rate of increase in that stock from period $t-1$ to period t can be written as the derivative of $V_{ij}(t)$, $\frac{d}{dt}\dot{V}_{ij}(t)$, or for convenience as $\dot{V}_{ij}$.[8]

We can now rewrite our basic input-output balance equation (system (2.4) in chapter 2) as follows:

$$X_i(t) - \sum_{i=1}^{n} a_{ij}X_j(t) - \sum_{j=1}^{n} (V_{ij}(t) - V_{ij}(t-1)) = Y_i(t)$$
$$i=1, 2, \ldots, n$$

or,

$$X_i(t) - \sum_{i=1}^{n} a_{ij}X_j(t) - \sum_{j=1}^{n} \dot{V}_{ij} = Y_i(t) \qquad (5.1)$$
$$i=1, 2, \ldots, n$$

Equation (5.1) may be interpreted as stating that:

The total output of industry i in year t, $(X_i(t))$, *minus* the sum total of all intermediate input flows of commodity i which serve the current production

[7] For a much more exhaustive analysis of the characteristics of the dynamic input-output system, see Leontief and others, *Studies in the Structure of the American Economy* (New York, Oxford University Press, 1953), chapter 3.

[8] Even though the derivative of $V_{ij}(t)$, V_{ij}, mathematically represents the 'change' in the stock of capital goods with respect to an 'instantaneous rate of change' in time, dynamic input-output models normally designate $\frac{dV}{dt}$ ij (t) as the change from one time period to the next, i.e. $V_{ij}(t) - V_{ij}(t-1)$. In effect, we are differentiating over one-period or one-year intervals to bring our mathematical model into conformity with the economic realities of development planning.

requirements of itself and the other n—1 industries in year t, $\left(\sum_{j=1}^{n} a_{ij}X_j(t)\right)$, *minus* the sum total of its output which is used to increase the capital stocks of itself and the other n—1 industries between years t—1 and t $\left(\sum_{j=1}^{n} \dot{V}_{ij}\right)$, is *equal* to the amount of good i available to satisfy the final demands of consumers, the government, and forcigners in year t.

Thus, while the second term of equation (5.1) represents the intermediate demand for 'current' material inputs (e.g. the textile industry needs raw cotton to produce its fabrics), the new third term describes those intermediate inputs which are absorbed 'on capital account' (e.g. a textile firm has to purchase cement to build a new plant so that it may expand productive capacity).

The set of structural flow equations, $(x_{ij}=a_{ij}X_j)$, describing the current inter-industry requirements of each sector of the economy must now be supplemented by a corresponding set of n^2 'structural capital stock' relationships:

(5.2) $$V_{ij}=b_{ij}X_j \qquad \begin{array}{l} i=1, 2, \ldots, n. \\ j=1, 2, \ldots, n. \end{array}$$

The $n^2 b_{ij}$'s are none other than the system's subsectoral 'stock' or 'capital' coefficients which were described in the previous section. Each b_{ij} represents that amount of good i *per unit* of output of good j that has to be purchased *by industry j* as capital goods over and above its purchases of material inputs $(a_{ij}X_j)$ in order to meet its (i.e. j's) total output targets. For example, if i stands for machine tools and j for bicycles, the subsectoral capital coefficient b_{ij} indicates the stock of machine tools required per unit of annual bicycle output. With b_{ij} known, the expression $V_{ij}=b_{ij}X_j$ would tell us what the total stock of machine tools (V_{ij}) must be in order that the local bicycle factory may produce an output of X_j bicycles per annum.

Just as we derived a square matrix 'A' of technical coefficients of production, we can likewise compute an n×n square matrix 'B' of subsectoral capital coefficients which would be written as follows:

$$B=\begin{bmatrix} b_{11} & b_{12} & \ldots\ldots\ldots\ldots & b_{1n} \\ b_{21} & b_{22} & & b_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ b_{n1} & b_{n2} & \ldots\ldots\ldots\ldots & b_{nn} \end{bmatrix}.$$

Each column of this matrix lists the individual capital requirements per unit of output of one particular sector of the economy. Thus, if sugar refining is

designated as industry 1, column 1 would list all subsectoral capital/output ratios of the sugar refining industry. Obviously, the summation of all elements in any one column, i.e. $\sum_{i=1}^{n} b_{ij}$, would equal the sectoral capital/output ratio of the industry represented by that column. Similarly, the global capital/output ratio, k, for the entire economy would be given by some weighted average of these n^2 sectoral capital/output ratios.

The overall balance between inputs and outputs expressed in (5.1) comprise, however, only *changes* in capital stocks rather than entire stocks themselves. We must therefore differentiate both sides of equation (5.2) with respect to time before we can transform these stock equations into relations between changes in specific capital stocks held by various industries, $\dot{V}_{ij}$, and changes in the rates of output of these industries, $\frac{dX}{dt}j$, equal to $X_j(t)-X_j(t-1)$ or, for convenience $\dot{X}_j$. Thus, we may say that

$$[V_{ij}(t)-V_{ij}(t-1)]=b_{ij}\ [X_j(t)-X_j(t-1)]$$

or

$$\dot{V}_{ij}=b_{ij}\dot{X}_j \qquad \begin{array}{l} i=1, 2, \ldots, n \\ j=1, 2, \ldots, n \end{array} \tag{5.3}$$

where $\dot{V}_{ij}$ represents the *net* induced purchase of new investment goods from industry i by industry j.

It is this step (5.3) which ultimately dynamizes our input-output system. If we now substitute this new capital relationship into (5.1), we can arrive at a final system of dynamic input-output equations:

$$X_i(t)-\sum_{j=1}^{n} a_{ij}.\ X_j(t)-\sum_{j=1}^{n} b_{ij}.\ [X_j(t)-X_j\ (t-1)]=Y_i(t).$$
$$\text{where } i=1, 2, \ldots, n$$

or, if put in matrix form the entire dynamic system may be expressed as:

$$\bar{X}(t)-[A].\bar{X}(t)-[B].[\bar{X}(t)-\bar{X}(t-1)]=\bar{Y}(t).$$

This dynamic input-output system can be described mathematically as a system consisting of n linear difference equations with constant coefficients, i.e. the a_{ij}'s and b_{ij}'s.[9] Its n 'unknowns' are the same as those of the corresponding static system, namely, the total outputs of all n goods and services, $X_1(t)$, $X_2(t), \ldots, X_n(t)$. Their mutual interdependence now also involves, however, the yearly *changes* in these total outputs, $\dot{X}_1, \dot{X}_2, \ldots, \dot{X}_n$. Given the values of total output for each industry in any base period, t_0, the solution of this dynamic system would yield a time-path portraying the maximum attainable rate of

[9] For a discussion of the solution to these first-order difference equations, see Caroline Dinwiddy, *Elementary Mathematics for Economists* (Nairobi, Oxford University Press, 1967), chapter XV.

growth of *each individual industry* in the economy when all of its repercussions with other industries have been accounted for. Once the time-path for the total output of a particular industry, say, the transport industry, has been determined, the Ministry of Planning could then compute a time-path of efficient capital formation by substituting the values for all n total outputs and solving that particular equation which describes the current and capital requirements of the transport sector. In an analagous procedure the dynamic input-output model can also be used to approximate the detailed time-path of intermediate capital import requirements,[10] anticipated government tax revenues, and skilled manpower necessities over the course of the development plan. In short, wherever the stage of development and the adequacy of statistical data permit the utilization of some variant of the dynamic input-output approach, then the adoption of this method would represent a considerable improvement in the methodology of arriving at a comprehensive and internally consistent development plan.

INPUT-OUTPUT AND THE PROBLEM OF STRUCTURAL CHANGE

Before we proceed to discuss 'activity analysis' which supplements the input-output methodology with the techniques of linear programming to investigate questions of feasibility and optimality in the development plan, we must be aware of one main aspect of the input-output model which is inconsistent with an important notion of the theory of economic development. This is the apparent dichotomy between the input-output assumption of a single-process production function with constant technical and capital coefficients in each sector of the economy and the notion of structural change as a concomitant phenomenon of the process of economic development. Two crucial planning questions emerge from this dichotomy. First, how are we to reconcile output and capital requirement projections based on existing technological relationships with the desire to introduce new methods of production and improved technologies? Second, how are we to consider and treat the establishment of entirely new industries, e.g. of the import-substituting variety, with the tools of interindustry economics?

The second problem presents no great conceptual or practical difficulties for it can be handled relatively easily within an existing input-output framework. As new industries emerge and commence engaging in intermediate material and capital transactions with other sectors of the economy, we merely fill in the so-called 'empty boxes' of our input-output table by adding another row of intermediate and final outputs and another column of material and factor inputs to represent the activities of this new industry. We could then compute

[10] Capital import requirements of a particular industry might be computed as the difference between the total amount of each type of capital-good needed minus that amount which can be supplied by local producers.

the relevant technical and capital coefficients of production either on the basis of preliminary engineering information about the structure of production in this new sector or on the basis of the statistical experience of similar industries already established in countries of comparable economic development. In addition, wherever the introduction of a new industry results in an altered pattern of interindustry transactions, the existing material and capital coefficients will have to be modified accordingly. But this leads us into the more difficult first question of structural change within established industries.

The expectation of significant structural changes, which are an inherent manifestation of the process of economic development, introduces a serious complication for the direct application of simple static and dynamic input-output techniques. Two related assumptions of input-output analysis give rise to this problem, namely (1) the seemingly unrealistic assumption (especially in long run analysis) that coefficients of production do not change over time, and (2) the highly restrictive assumption that each industry has at its disposal only one method or 'process' (reflected by a single set of technical coefficients) of producing its particular commodity.

The problem of variable production coefficients over the course of time can ordinarily be handled within the existing model by means of a periodic statistical revision (say every three years) of the empirical input-output table. Alternatively, if such periodic revisions are beyond either the financial or the research capabilities of government statistical agencies, planners might still be able to project future coefficients by using technological data from other slightly more developed countries or by merely extrapolating from past trends. Finally, the Ministry of Planning should always keep a close watch on certain crucial industries like transportation and construction where any change in production techniques or purchasing patterns can have significant repercussions throughout the entire economy.

Unlike the problem of variable technical and capital coefficients, however, the problem of multi-process production possibilities within individual industries presents serious analytical and practical difficulties for the application of most simple input-output manipulations. Suppose, for example, that there exist two alternative methods of producing the same output of, say, cotton textiles. The textile industry would consequently have *two* different sets of technical coefficients of production. The problem then arises as to which process to use under various circumstances. This is a problem not only of equating inputs with outputs to ensure the internal consistency of the development plan but, more importantly, a problem of choosing the *optimal* production process, i.e. the one that can yield the same total output while using relatively less of whatever resources are most limited. In short, the existence of alternative technologies and resource 'constraints' requires use of the more elaborate methods of linear programming and the formulation of a combined input-output linear programming model, known in the literature as 'activity analysis'.

ACTIVITY ANALYSIS AND THE NOTION OF OPTIMIZATION IN DEVELOPMENT PROGRAMMING

FROM INPUT-OUTPUT TO ACTIVITY ANALYSIS

While the formulation of a development plan on the basis of the dynamic input-output model greatly aids in co-ordinating the diverse production activities of many different industries within a unified and internally consistent analytic framework, economic planners in less developed countries are inevitably faced with the additional problem that although a given development plan might be free from inner contradictions, it nevertheless requires more resources than will be conceivably available during the planning period. The so-called 'boundary conditions' or 'constraints', which are commonly handled with the tools of linear programming at the micro-economic level, are also of great importance when one attempts to plan the activities of an entire economy. In a typical linear programming situation a firm or farm must not only avoid producing beyond its resource capabilities but it must also often choose from among a number of possible methods of production that method which makes the best use of its limited resources.[11] This is also the case in comprehensive development programming where there often exist alternative methods of producing different outputs. Given, for example, a country's capital, labour, and foreign exchange constraints, planners must attempt to choose both *optimal* production techniques and *optimal* paths of capital accumulation. Thus, although the establishment of a steel factory might make a significant contribution to GDP, such a project will normally require a great deal of capital. If capital scarcity represents the main operational constraint in this economy, then perhaps the capital funds that would have been required to build the steel plant might be more efficiently utilized in, say, agricultural and/or light manufacturing projects. Activity analysis and programming represents an attempt to supplement the interindustry model with the question of 'choice' both between methods of supply and the composition of final demands. It also attempts to recognize strategic resource constraints that must be taken into account when formulating a development programme.

THE ACTIVITY ANALYSIS MODEL

Basic Concepts: Activities and Restrictions

An activity may be defined as any possible transformation of fixed proportions of commodity inputs into fixed proportions of commodity outputs. In our static

[11] For a description of linear programming as it relates to the theory of producer behaviour, see P. W. Bell and M. P. Todaro, *Economic Theory: An Integrated Text with Special Reference to Tropical Africa and Other Developing Areas* (Nairobi, Oxford University Press, 1969), chapter 4.

and dynamic input-output models each industry in the economy was assumed to utilize only one activity or technological production process which could be represented by a column vector from the Leontief [I—A] matrix.[12] For example, we could represent the activity vector of, say, the textile industry denoted as sector 3 in a six-industry model, as

$$A_3 = \begin{vmatrix} -a_{13} \\ -a_{23} \\ +a_{33} \\ -a_{43} \\ -a_{53} \\ -a_{63} \end{vmatrix}$$

where a positive coefficient denotes an output and a negative coefficient an input. In general, therefore, we can represent any activity j as a column vector:

$$A_j = \begin{vmatrix} a_{1j} \\ a_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{mj} \end{vmatrix} .$$

Note that if industry j has at its disposal, say, two possible activities or production processes, they could be written as:

$$A_j = \begin{vmatrix} a_{1j} \\ a_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{mj} \end{vmatrix} \quad \text{and} \quad A^1_j = \begin{vmatrix} a^1_{1j} \\ a^1_{2j} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a^1_{mj} \end{vmatrix} .$$

[12] As we shall now see, the only difference between a column vector in the [I—A] matrix and its counterpart in activity analysis is that in the latter model the diagonal element $(1-a_{ii})$ becomes simply a_{ii}, with a positive sign.

If we now combine all possible activities of every producing sector in the economy we can derive the following *technological activity matrix:*

$$[A]=(A_1A_2\ldots\ldots A_n)=\overbrace{\begin{bmatrix} a_{11}a_{12} & \ldots\ldots & a_{1n} \\ a_{21}a_{22} & \ldots\ldots & a_{2n} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ a_{m1}a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix}}^{\text{n activities}} \left.\vphantom{\begin{bmatrix} a \\ a \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a \end{bmatrix}}\right\} \text{m commodities (including primary inputs)}$$

This technological activity matrix need not necessarily be a 'square' matrix since where there exists some choice among alternative activities, the number of activities, n, will be *greater* than the number of commodities, m.[13]

In addition to a matrix of activities, the activity analysis model also includes a set of autonomous elements called *restrictions* or *constraints* which are assumed to be given. These constraints normally include final demands, primary input supplies, and, in the case of most African countries, foreign exchange. There will be a restriction on the production of each type of commodity; positive for final target outputs, negative for factor inputs, and zero for intermediate commodities (i.e. total supply must equal total demand). The set of m restrictions may also be written as a column vector:

$$B=\begin{vmatrix} B_1 \\ B_2 \\ B_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ B_m \end{vmatrix}.$$

Since there will be one restriction on each commodity and primary input, the total number of restrictions, m, will equal the combined number of commodities and primary inputs, m.

[13] Note that in the activity analysis model a primary input, say labour, is also considered a commodity whose output (or use) is subject to a boundary constraint.

The Programming Problem

The problem to be solved with the activity analysis model is quite analagous to the simple linear programming problem, namely, to *maximize (or minimize) some linear objective function of activity levels subject to all the final output, primary input, and intermediate commodity restrictions.* Formally, the programming problem may be represented as follows:

$$\text{maximize (or minimize) } Z = \sum_{j=1}^{n} z_j X_j.$$

Subject to

$$\sum_{j=1}^{n} a_{ij} X_j \geq B_i \quad (i = 1, 2, \ldots, m)$$

where

$$X_i \geq 0 \qquad (j = 1, 2, \ldots, n).$$

'Z' may be total output whose subjective value is to be *maximized* in which case each z_j would equal the value which planners attach to one unit of output of commodity j. Alternatively, if 'Z' represents total cost and the problem is one of *minimization*, then each z_j might represent either the actual or 'opportunity' cost of producing a unit of output of commodity j.

We can now usefully distinguish four main differences between the equations of the above activity analysis model and those which make up the input-output system:

1. There are a number of feasible solutions to our objective function but we seek to choose the best or *optimal* solution to our problem. In the input-output model there exists no similar criterion for optimality.
2. The activity model lists alternative ways of producing the same output while the input-output model assumes a single process production function for each industry.
3. Unlike the static or dynamic input-output model, the primary factors (land, labour, and capital) are as much an inherent part of the activity model as are the produced commodities, since any feasible solution must satisfy resource constraints as well as final use requirements.
4. Since each restriction consists of an inequality, the activity model provides for the possibility of non-use of some resources. Similarly, its formulation of the development problem allows for the optimal capacity use of the more limited resources.

A Simple Graphical Example

To illustrate the central idea of activity analysis, let us consider the following very simple interindustry case for Malawi. We shall assume that Malawi produces only two commodities (say, manufactured goods X_1, and agricultural

goods X_2) using two activities[14] (A_1 and A_2) and two primary inputs (labour and capital). Suppose that in order to produce one extra unit of its output the manufacturing sector must purchase 0.25 units of agricultural goods, 7.5 units of labour, and 1.25 units of capital. The 'activity vector' for industry 1 could then be represented as:

$$A_1 = \begin{vmatrix} 1.00 \\ -0.25 \\ -7.50 \\ -1.25 \end{vmatrix} .$$

Similarly, suppose that in order to produce one additional unit of its output, the agricultural sector must purchase 0.5 units of manufactured goods and use 5.0 units of labour and 2.5 units of capital.[15] The activity vector for agriculture, therefore, would be:

$$A_2 = \begin{vmatrix} -0.5 \\ 1.0 \\ -5.0 \\ -2.5 \end{vmatrix} .$$

Let us also assume (1) that development planners in Malawi require the minimum increase in the final demand for manufactured and agricultural goods over the next period to be +10 and+50 units respectively, i.e. $Y_1=+10$, $Y_2=+50$, and (2) that we can expect no more than 600 additional units of labour and 200 units of capital to become available during this planning period. Consequently, our set of restrictions could be written as follows:

$$B = \begin{vmatrix} +\Delta Y_1 \\ +\Delta Y_2 \\ -\Delta L \\ -\Delta K \end{vmatrix} = \begin{vmatrix} +\ 10 \\ +\ 50 \\ -600 \\ -200 \end{vmatrix} .$$

Suppose, finally, that Malawi wishes to maximize the possible *increase* in the value of its total output (ΔZ) where the marginal value of a unit of both manufacturing and agricultural output (i.e. z_1 and z_2) is 1.0.

[14] For the present, we are assuming that there is only one method of producing X_1 and X_2. This assumption will be dropped shortly.

[15] Note that in our example we have made the production of *additional* agricultural goods relatively capital intensive and *additional* manufactured goods relatively labour intensive. This may not be as implausible as it first appears since we are considering additional output over, say, the next five year period. In such circumstances it is not so unlikely that a relatively greater amount of capital will be devoted towards improving the productivity of the agricultural sector in order to release enough labourers to work in the sector that produces light labour-intensive manufactured goods like soap, textiles, etc.

Combining the activities, commodities, and restrictions, we can arrive at the following statement of the activity analysis problem:

maximize $\Delta Z = 1.0 \Delta X_1 + 1.0 \Delta X_2$.

Subject to

(1) $\quad 1.0 \ \Delta X_1 - 0.5 \ \Delta X_2 \geq +10$ final demand (ΔY_1) constraint

(2) $\quad -0.25 \ \Delta X_1 + 1.0 \ \Delta X_2 \geq +50$ final demand (ΔY_2) constraint

(3) $\quad -7.5 \ \Delta X_1 - 5.0 \ \Delta X_2 \geq -600$ labour (ΔL) constraint

(4) $\quad -1.25 \ \Delta X_1 - 2.5 \ \Delta X_2 \geq -200$ capital (ΔK) constraint

where,

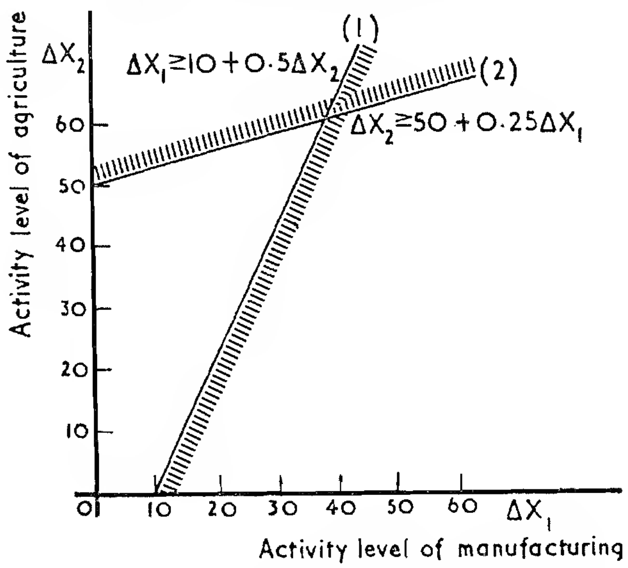
$\Delta X_1 \geq 0$ and

$\Delta X_2 \geq 0$.

A graphical solution to this problem can be arrived at using the following procedure:

Figure 5.1

Final Demand (Use) Constraints



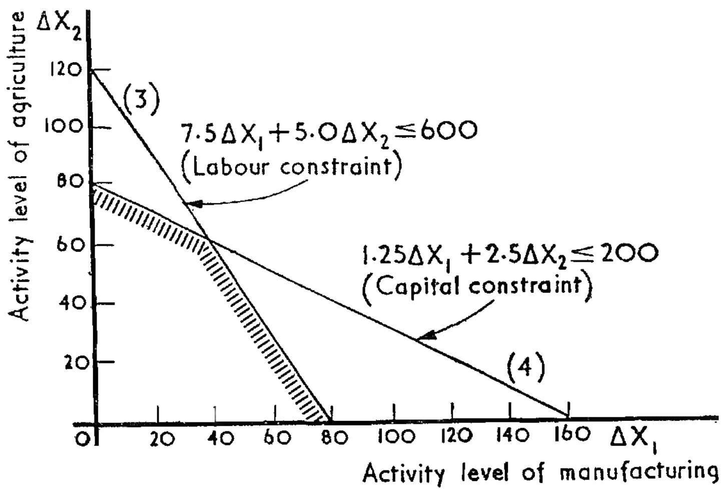
We first plot the inequalities representing the two final demand constraints. Since a minimum of 10 additional units of manufacturing output must be produced, constraint line (1) intersects the ΔX_1 axis at $+10$. All other manufacturing output will be produced to satisfy the intermediate demands of agriculture. Since each extra unit of agricultural output requires 0.5 units of manufactured goods, the slope of line (1) will be given by:

$$\text{slope of manufacturing constraint} = \frac{\text{the change in } \Delta X_2}{\text{the change in } \Delta X_1} = \frac{1.0}{0.5} = +2.$$

Thus the additional production of manufactured goods when both intermediate and final demands are considered will be represented by any point to the right of line (1). Any point to the left of line (1) would either violate the final demand requirement of +10 or be inconsistent with the additional output of industry two. Similarly, since 50 units of additional agricultural goods are required to satisfy minimum final demand requirements, the intercept of constraint line (2) on the ΔX_2 axis will be +50 and its slope will be equal to +0.25. All points above line 2 are feasible while any point below this line would either violate the final demand requirement or be inconsistent with the ΔX_1 level of output. It is evident from this graph that only those combinations of ΔX_1 and ΔX_2 which lie on or within the boundary of the shaded area represent feasible levels of additional manufacturing and agricultural output.

Figure 5.2

Resource Constraints

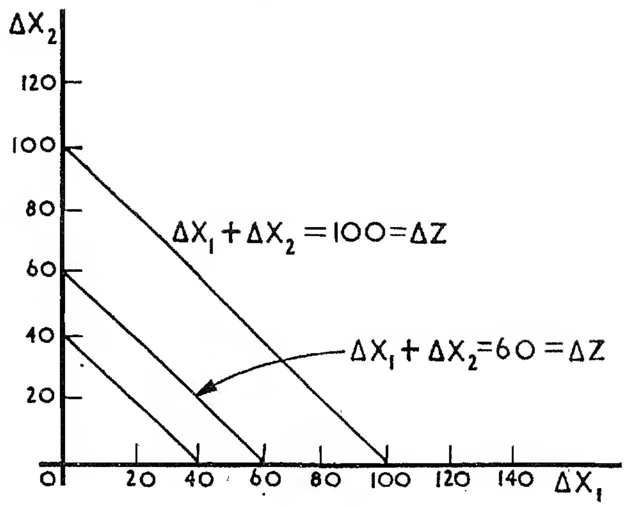


Using the same axes, we can now plot the labour and capital constraint lines derived from restrictions (3) and (4). Any combination of ΔX_1 and ΔX_2 that lies outside of the shaded area of this second graph (5.2) would represent a combination of manufacturing and agricultural output that requires more of one or both resources than will be available.

The objective function can be represented as an infinite number of parallel lines each of which intersects both the ΔX_1 and ΔX_2 axes at equal points. Thus, for example, the outermost line in the graph below would represent a locus of all combinations of ΔX_1 and ΔX_2 that yield a value of 100 (e.g. $100\Delta X_1$ and $0\Delta X_2$, $60\Delta X_1$ and $40\Delta X_2$, etc.) while the innermost line yields a value of 40.

Figure 5.3

Objective Function



The Optimal Solution

By combining graphs (5.1) to (5.3) into a single diagram we can arrive at a graphical solution to our activity analysis problem which will reveal that combination of ΔX_1 and ΔX_2 which will maximize the possible increase in the value of total output in Malawi given our resource and final use restrictions. It will be seen from the following graph (5.4) that if Malawi produces 40 additional units of manufactured goods and 60 more units of agricultural products, additional output value will be maximized and all extra resources will be fully utilized in an optimal way.[16]

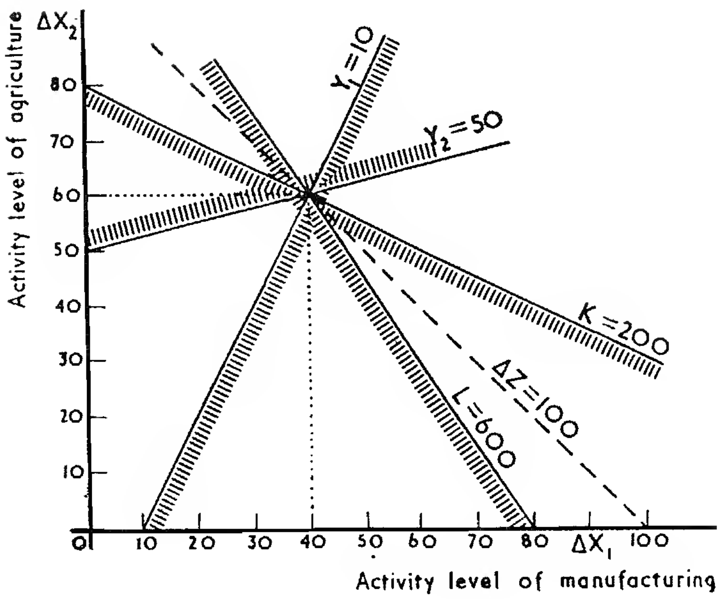
INTRODUCING CHOICE INTO ACTIVITY MODELS

In the graphical example we combine one notable feature of the activity analysis model with the basic interindustry framework, namely, the notion of optimal resource allocation. However, by endowing each sector with only one activity or production process and by assuming a minimum level of final output for each sector this simple model did not deal with another notable feature of activity analysis, i.e. the notion of choice on (a) the supply side and (b) the demand side. We will analyse each of these phenomena separately in the remainder of this chapter.

[16] Note that due to the special numerical values chosen in this simplified programming problem, the optimal solution turns out also to be the only possible feasible solution. Can you verify this from the graph?

Figure 5.4

The Optimal Solution



Choosing from Alternative Activities for a Single Output

Since the problem of choosing an optimal pattern of supply in the context of a comprehensive development plan, when different outputs can each be produced with a variety of methods, would present a most formidable task of mathematical programming that would take us far beyond the scope of this present chapter, we will merely attempt here to represent the problem for a single industry. Hopefully, the intuitive analysis of choice among alternative processes when there are many industries will be grasped from this simplified one-product example.

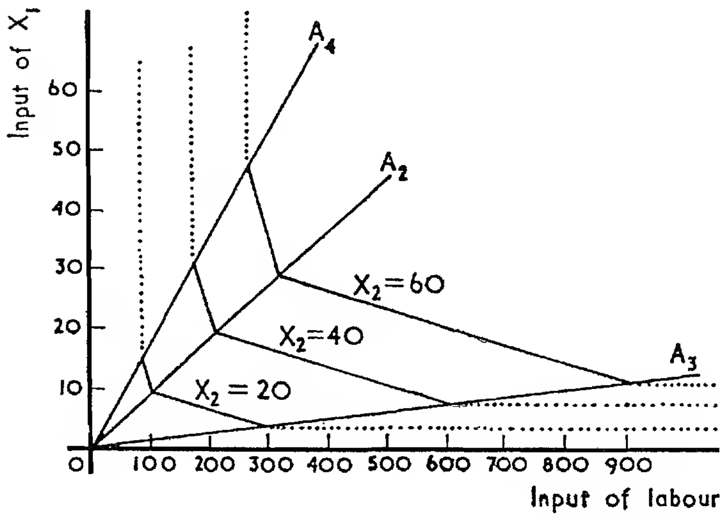
Suppose, for example, that Malawi's agricultural sector had available not one but *three* activities for producing more of its output X_2. The activities are represented by the following vectors:[17]

$$A_2 = \begin{vmatrix} -0.5 \\ 1.0 \\ -5.0 \end{vmatrix} \quad A_3 = \begin{vmatrix} -0.2 \\ 1.0 \\ -15.0 \end{vmatrix} \quad A_4 = \begin{vmatrix} -0.8 \\ 1.0 \\ -4.0 \end{vmatrix}.$$

[17] To facilitate two-dimensional graphical analysis we have omitted capital coefficients and thus are assuming only one variable resource, labour.

Since two new activities are now available (i.e. A_3 and A_4), we can represent the production function of the agricultural sector in terms of a set of production isoquants showing these three alternative ways of producing the same level of output. To construct an isoquant map for agriculture, X_2, we can use the two inputs (X_1 and L) as axes and compute for any level of output X_2 the required amounts of X_1 and L in terms of each of the three activities. If we then connect these three combinations of X_1 and L that yield an equal output of X_2 we will have constructed the isoquant for this output level. For example, if X_2 is 20 then activity A_2 dictates that 10 units of X_1 (i.e. $X_{ij}=a_{ij}X_2$ or $X_{12}=0.5\times20=10$), and 100 units of labour (i.e. $L_j=l_jX_j$ or $L_2=5.0\times20=100$) will be required by industry 2 to produce these 20 units of output. If activity A_3 is used, then 4 units (i.e. $0.2\times20=4$) of X_1 and 300 units (i.e. $15\times20=300$) of labour will be needed. In the following diagram (5.5) the agricultural isoquants representing 20, 40 and 60 units of X_2 output are plotted on the basis of activities A_2, A_3, and A_4.

Figure 5.5



Now, in our single graphical example of the previous section where only activity A_2 was available to agriculture, everything worked out very neatly. But what if there actually were three agricultural activities from which planners in Malawi have to choose and the combined choice of A_1 and A_2 turns out to leave us with some excess labour? Perhaps we could still get 60 units of additional agricultural output and *more* manufacturing output by using A_3 under other circumstances! The student will fully appreciate the complexity of efficient development programming when he realizes that development planners must attempt to choose the optimal pattern of supply from a great variety of products

each of which is capable of being produced with one or more techniques of production.[18]

Choosing the Optimal Pattern of Final Demands

One of the principal limitations of input-output analysis in the formulation of development programmes is that the planned product-mix of final demand or total output must be specified before calculating material and resource requirements. Thus, if we are given alternative sets of final demand targets we can then use the input-output methodology to determine internally consistent product and capital flows. But since the input-output system contains no element of *choice* within the model, there exists no internal mechanism to arrive at an *optimal* set of final demand targets under given resource and material supply conditions. By using the techniques of activity analysis, however, the question of finding an optimal solution to the programming problem can be approached systematically without having to go through a great number of alternative interindustry calculations.

Suppose we modify our simple two-sector graphical problem for Malawi by adding a third constraint representing the maximum level of *foreign exchange* that is expected to be available over the next planning period and put the problem in terms of total rather than additional output. Specifically, let us assume that the following foreign exchange constraint is operative in the Malawi economy:

$$1.0X_1+0.01X_2\leq 200 \text{ (units of foreign exchange)}$$

or, in terms of our 'activity analysis' formulation,

$$-1.0X_1-0.1X_2\geq -200.$$

Furthermore, rather than specifying a minimum required level of final output for manufacturing (X_1) and agriculture (X_2), we seek to determine that combination of final demands which will maximize the total value of *final* output of these two commodities when X_1 is given a per unit value of 1.2 and X_2 a value of 1.0.[19] Using the same (I$-$A) matrix and resource constraints as in the graphical example, we can now express our problem in terms of the linear programming format with slack variables. Thus, we have the following:

$$\text{maximize } Z=1.2Y_1+1.0Y_2.$$

Subject to

(5.4) $\quad 1.0X_1-0.5X_2-Y_1=0$ (supply=demand constraint)

(5.5) $\quad -0.25X_1+1.0X_2-Y_2=0$ (supply=demand constraint)

[18] It should be noted in passing that activity analysis may also be used to handle other aspects of choice on the supply side, e.g. (1) choice between imports and domestic production, i.e. import substitution; (2) choice between current production and inventory depletion.

[19] These values may either be expressed as actual prices or relative planning priorities, i.e. planners deem one unit of manufacturing output as being worth 20 per cent more than a unit of agricultural output in terms of the overall objectives of the development plan.

(5.6) $-7.5X_1 -5.0X_2-S_a = -2{,}000$ (labour constraint)
(5.7) $-1.25X_1-2.5X_2-S_b = -600$ (capital contraint)
(5.8) $-1.0X_1 -0.1X_2-S_c = -200$ (foreign exchange constraint)

$$S_a \geq 0 \quad S_b \geq 0 \quad S_c \geq 0$$
$$X_1 \geq 0 \quad X_2 \geq 0 \quad Y_1 \geq 0 \quad Y_2 \geq 0.$$

Suppose for comparative purposes that we start with $Y_1=10$ and $Y_2=50$ as in the graphical example, so that $X_1=40$, $X_2=60$, i.e. substituting $Y_1=10$ and $Y_2=50$ into equations (5.4) and (5.5) above and solving, we get:

(5.9) $X_1-0.5X_2=10$
(5.10) $0.25X_1+1.0X_2=50$

or, $2.00X_1-1.0X_2=20$
$-0.25X_1+1.0X_2=50$

$1.75X=70$
$X_1 = 40$
$\therefore$ $X_2 = 60.$

Substituting these values for X_1 and X_2 into resource constraints (5.6), (5.7) and (5.8) we discover that the amounts of *unused* resources will be:

$$S_a=2{,}000- 7.5X_1-5X_2 = 1400 \text{ (unused labour)}$$
$$S_b= 600-1.25X_1-2.5X_2 = 400 \text{ (unused labour)}$$
$$S_c= 200- 1.0X_1-0.1X_2 = 154 \text{ (unused foreign exchange).}$$

Similarly, the value of final output is:

$$Z=1.2(10)+1.0(50)=62.$$

Surely, with all of this excess capacity we can do much better than the above combination of final demands? Consider the following graphical portrayal of this problem.

It is obvious from the diagram (5.6) that a final demand combination of 10 units of Y_1 and 50 units of Y_2 which requires a total output of 40 units of X_1 and 60 units of X_2 lies far within the area of feasible solutions and consequently cannot be an optimal solution.[20]

Solution using the Simplex Method

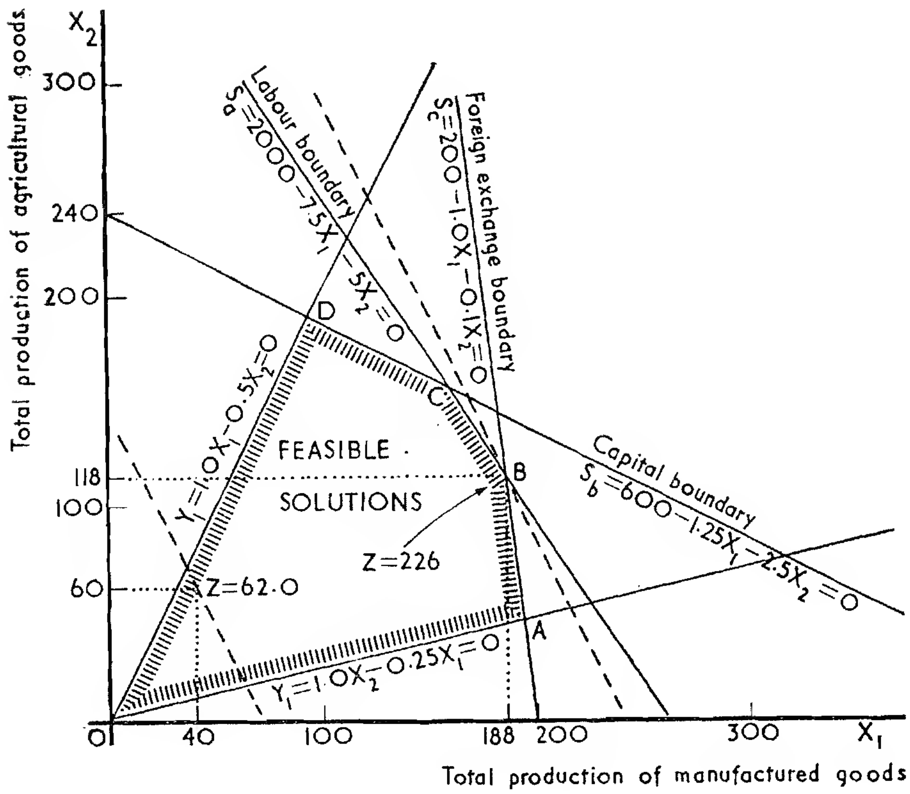
By substituting equations (5.4) and (5.5.) into the objective function, we can express the maximization problem in terms of total outputs which will in turn yield a solvable linear programming problem with two choice variables (X_1 and X_2) and three constraints (equations (5.6), (5.7) and (5.8)). Thus, our new objective function becomes:

$$\text{maximize } Z=1.2\ (1.0X_1-0.5X_2)+1.0(-0.25X_1+1.0X_2)$$

[20] Note that the optimal solution will always lie on a corner point of the boundary of feasible solutions, i.e. at either 0, A, B, C, or D in Figure 5.6. Thus, the techniques of linear programming make the choice of an *optimal* solution relatively easy.

Figure 5.6

Locating the Optimal Solution to an Activity Analysis Problem



which reduces to,

maximize $Z=0.95X_1+0.40X_2$

where the coefficients 0.95 and 0.40 now refer to the 'value added' (money) in producing each of the two commodities. Graphing this new objective function into the above diagram we see that our optimal solution will be given by point B where $X_1=188$ and $X_2=118$ and $Z=226$. The corresponding optimal combination of Y_1 and Y_2 will be 129 and 71 respectively. But normally to obtain a solution we must use the 'simplex' procedure which requires our linear programming problem to be reformulated as:

maximize $Z=0+.95X_1+.40X_2$.

Subject to

$S_a=2{,}000-7.5X_1\ -5X_2$

$S_b=\ 600-1.25X_1-2.5X_2$

$$S_c = 200 - 1.0X_1 - 0.1X_2$$
$$S_a \geq 0 \quad S_b \geq 0 \quad S_c \geq 0$$
$$X_1 \geq 0 \quad X_2 \geq 0.$$

We can now solve for the optimal combination of X_1 and X_2 (and thus, for Y_1 and Y_2 also) by using the simplex methodology.[21] Thus,

MATRIX 1

	Z	X_1 ↓	X_2	$-\frac{\text{Col. (1)}}{\text{Col. (2)}}$
Value	0	.95	+.40	
S_a	2,000	−7.5	−5.0	267
S_b	600	−1.25	−2.5	480
S_c	200	−1.0*	−0.1	200←

MATRIX 2

	Z	S_c	X_2 ↓	$-\frac{\text{Col. (1)}}{\text{Col. (3)}}$
Value	190	−.95	+.305	
S_a	500	7.5	−4.25*	117.6←
S_b	350	1.25	−2.375	147.9
X_1	200	−1.0	−0.1	200.0

MATRIX 3

	Z	S_c	S_a
Value	226	− .41	−.072
X_2	117.6	1.76	−.235
S_b	70.6	−2.9	+.559
X_1	188.3	−1.18	+.026

The negative coefficients in row (1) of matrix 3 tell us that we have reached our optimal solution. The simplex method thus verifies our graphical observation that the total value of final output will be maximized at approximately

[21] See Bell and Todaro, op. cit., chapter 4, for a description of the methodology of the simplex solution.

226 by producing approximately 118 units of agricultural goods and 188 units of manufactured goods (point B in the diagram) using all available labour and foreign exchange (i.e. S_a and S_c=0) and having approximately 70 units of capital left over. Substituting X_1=188 and X_2=118 into our supply=demand equations (5.4) and (5.5), we can solve for the optimal combination of Y_1 and Y_2. Thus,

$$Y_1=1.0\ (188)-0.5\ (118)=129$$
$$Y_2=0.25(188)+1.0(118)=71.$$

The student should check his understanding of this problem and the methodology of activity analysis by ascertaining what the optimal solution and the resultant resource use would bc if the per unit final demand values of manufacturing and agriculture were reversed, i.e. z_1=1.0 and z_2=1.2.

It should be obvious from the above presentation that choice of relative priorities as reflected in the assigned values for z_i will greatly affect the optimal pattern of production and factor utilization of any comprehensive development plan. But it takes a detailed model with the sophistication of the interindustry framework in general, and the mathematical techniques of activity analysis in particular, to ascertain fully the entire range of economic implications of alternative targets and development priorities. If the student has grasped this very important concept of economic interdependence, he will have gone a long way down the road towards a more complete understanding of the nature and intricacies of economic planning.

Suggested Readings

R. A. Bishop, 'Input-Output Work as a Basis for Development Planning' (*Monthly Bulletin of Agricultural Economics and Statistics*, May 1956).

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Throughout the less developed nations of the world the quest for rapid economic progress has been predicated largely upon the formulation and implementation of comprehensive development plans. The primary purpose of this book is to introduce undergraduate students in developing countries to the most common and widely used economic models of development planning in poor nations. A secondary objective is to give students a feel for the 'thought processes' involved in the formulation of a development plan by actually working step by step through hypothetical planning problems. The emphasis is on plan formulation rather than plan implementation. Particular stress is placed on the input-output model, both static and dynamic, as a tool of comprehensive and internally consistent planning. No mathematical background beyond elementary algebra is required. In fact, the vast majority of examples utilize nothing beyond simple arithmetic techniques.

This short book is expected to make a useful contribution to undergraduate courses in Economic Development and Development planning. Its brevity and relative simplicity can also make it a useful reference work for the non-academic reader.

Prior to attending Yale University, from which he received his Ph.D. in Economics in 1967, Dr. Michael P. Todaro spent a year in Uganda as a Visiting Lecturer in the Department of Economics at Makerere University. After teaching for a year at Yale, Dr. Todaro returned to East Africa in 1968 where he spent two years as a Research Fellow at the Institute for Development Studies in Nairobi, during which time he also taught Economics at the University of Nairobi. He is currently an Assistant Director for Social Sciences at The Rockefeller Foundation in New York.

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